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DEPARTMENT OF CIVIL ENGINEERING

CIVIL ENGINEERING SYSTEMS LABORATORY

HIDECS 3:
FOUR COMPUTER PROGRAMS FOR THE HIERARCHICAL DECOMPOSITION OF SYSTEMS WHICH HAVE AN ASSOCIATED LINEAR GRAPH

by

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INTRODUCTION

Confronted by a system of elements, some linked to one another, others not linked, the most natural question to ask about the system is: "What are its subsystems".

The question is by no means a simple one; first of all, it all depends on what you mean by "subsystem". Each of the four programs described in this report is based on a different conception or definition of a "subsystem".

These programs, each on the basis of a different definition of 'subsystem', determine the decomposition of any system into its subsystems. In every case the system is taken to be defined by a set of elements, M, and a set of two-element links, L. The system is therefore completely described by a graph G(M,L); and is represented in the computer by a binary matrix.

The input, machine representation, and output, for all four programs are the same as in the program HIDECS 2, described in a previous report in this series. All the supporting subprograms like GENER, COUNT, CNVRT, etc, are also the same as in HIDECS 2. Each of the programs to be described in this report, namely BLDUP, STABL, SIMPX, EQCLA, is a core subprogram, under the control of MAIN thus similar in content and function to LGRMN, the core subprogram of HIDECS 2.

The actual system decompositions defined by these programs is however quite different; each one is intended to take care of certain weaknesses in HIDECS 2. So that this report may be selfcontained, we begin with a brief summary of HIDECS 2.

In HIDECS 2 each element of M is assumed to be a binary stochastic variable. The decomposition of the system into subsystems is specified by a tree of subsets of M. At the top level of the tree is the set M. At the second level there are two disjoint subsets of M, whose union is M. At each new level in the tree, every subset is broken into two further disjoint subsets. The problem is to find that tree which is most appropriate in view of the linkages defined by L, and which thus succeeds in isolating what we should most want to call the subsystems of the system.

It has been shown (Alexander, 1963a) that one way of doing this is to define the information transfer between sets of variables. At each level of the tree, a set of variables is broken into those two of its subsets with minimum information transfer between them. Let m be the number of variables in M, ℓ the number of links in L, s_1 and s_2 the numbers of variables in the two subsets ($s_1 + s_2 = m$), and r the number of links between the two subsets. The information transfer, corrected for bias toward special partitions, is:

$$\text{INFO} = \frac{\left\{ r - \left[\frac{2\ell}{m(m-1)}\right] s_1 s_2 \right\}^2 \delta}{s_1 s_2 \left[\frac{m(m-1)}{2} - s_1 s_2\right]}$$

, where δ is +1 or -1 according as the top bracket is positive or negative,

The program HIDECS 2 uses this function, INFO, as the criterion for a steepest-ascent hill-climbing procedure. The program begins by generating a random pair of complementary subsets of M. It then tests all pairs of subsets which can be derived from this pair by shifting a single element from one subset to the other. The best pair of subsets replaces the starting pair. The program repeats this process of testing and replacement until it finds a pair of subsets which cannot be improved by shifting a single element. Experiments show that the number of independent hill-climbs required to reach absolute optimum is small.

HIDECS 2 has three important weaknesses:

1. The fact that the decomposition is made in a series of binary steps leads to certain 'mistakes', since the holistic relatedness of system and subsystems is not properly taken into account.
2. The fact that the decomposition criterion INFO is based on very stringent assumptions about the nature of the system G(M,L). Namely, that the elements of M are binary

variables, that the two variable correlations are very small, and that the many variable correlations vanish altogether. These assumptions make it hard to find systems in the real world which the formalism of HIDECS 2 can adequately represent.

3. The fact that the subsets of elements which make the most natural subsystems of a system are not always disjoint, but often overlap.

In the four programs to be described, these weaknesses are overcome as follows:

In BLDUP, the decomposition criterion, though still essentially the same as that used in HIDECS 2, has been extended so that not only 2-way, but 3-way, 4-way, etc. partitions can all be compared with one another. This means that the decomposition into subsystems need not be defined stepwise, but can be defined all at once, and the holistic nature of the system thereby better preserved.

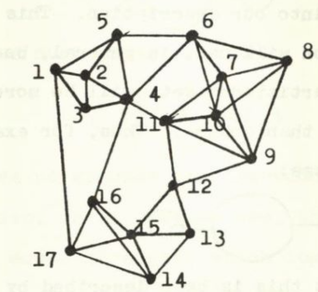
In STABL, SIMPX, and EQCLA the elements of the system are no longer assumed to be binary variables, or indeed variables at all. The elements of M may be elements of any kind, and the links between elements, though still only permissible between two elements at a time, may be of any sort whatever. In all three cases, as in BLDUP, the subsystems are defined simultaneously, not sequentially.

Finally, in SIMPX and EQCLA, the subsystems are defined in such a way that they overlap instead of being disjoint.

In fact, in these two cases the decomposition, instead of being a tree, is a lattice.

In the following four sections we shall examine the four subprograms BLDUP, STABL, SIMPX, EQCLA, in detail.

So that the content of each of the four programs may become intuitively clear, each section will close with decomposition of the following graph:

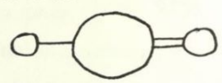


The deoomposition which HIDECS 2 gives of this graph, is as follows:

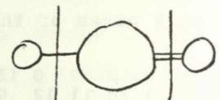
```
 1  2  3  4  5  0  0  0  0  0  0  0 13 14 15 16 17
 0  0  0  0  0  6  7  8  9 10 11 12  0  0  0  0  0
NEW LEVEL OF HIERARCHY
 1  2  3  4  5  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0 13 14 15 16 17
 0  0  0  0  0  0  0  0  0  0 11 12  0  0  0  0  0
 0  0  0  0  0  6  7  8  9 10  0  0  0  0  0  0  0
NEW LEVEL OF HIERARCHY
 0  2  0  4  5  0  0  0  0  0  0  0  0  0  0  0  0
 1  0  3  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0 14 15 16 17
 0  0  0  0  0  0  0  0  0  0  0  0 13  0  0  0  0
 0  0  0  0  0  0  0  0  9  0  0  0  0  0  0  0  0
 0  0  0  0  0  6  7  8  0 10  0  0  0  0  0  0  0
```

BLDUP

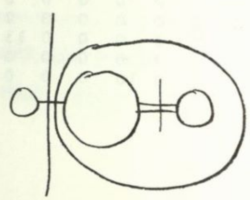
Before describing an n-way partition of M, let us explore the theory of achieving multiple decomposition by successive binary partitions. The purpose of partitioning the set M is to obtain as complete a description of the system's structure as we can. If we make a practice of partitioning into n subsystems at each step, we should, in general, introduce spurious regularities into our description. This is because the best n-way partition will not, in general, be entirely regular, some of the partitioned sets will be more strongly related to one another than others. Thus, for example, take the following simple case:



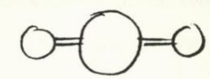
It might be argued that this is best described by a 3-way partition. Thus:



But the left-hand subsystem is less strongly connected to the center than the right hand one. If we described this by means of a 3-way partition we should be ignoring this assymetry or inequality. We shall have a more accurate picture of the structure if we describe it by means of two 2-way partitions. Thus:

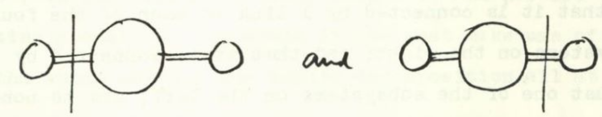


This kind of description fails only in that case where the linkages are exactly equal, as in:



In this case we must write

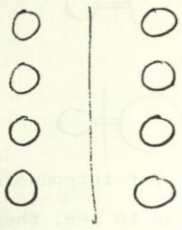
because we have no grounds for introducing any assymetry. But in this case, as is easy to see, there will be two 2-way partitions of equal strength, which together divide the set into 3 subsets. Thus:



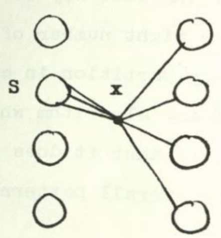
Indeed, it is not hard to convince oneself that in every case where an n-way partition is the best way to describe a decomposition, there are just the right number of equal best 2-way partitions to give that n-way partition in concert.

However, the defect of any algorithm which partitions M into two subsets at a time, is that it does not pay sufficient attention to the gestalt, or overall pattern of the subsystems, and therefore introduces a bias which by any reasonable intuition is a 'mistake'.

Consider an example. Suppose we have a system which, at the first level of decomposition, is to be divided into two subsystems, as shown below, leaving a situation which further partitioning will then split further into the four subsystems on the left, and four subsystems on the right.



Suppose now that there is one element x which we are in doubt about how to place in this first partition. Let us say that it is connected by 1 link to each of the four subsystems on the right; and that it is connected by 3 links to just one of the subsystems on the left, and to none of the others on the left.



Other things being equal, a two-way partition algorithm will, at this level of decomposition, assign this point to the right hand subsystems rather than to the left hand - because it is linked to the right by a total of 4 links and by a total of only 3 links to the left. Yet the outcome of this decision, when the decomposition proceeds to lower levels of decomposition, is that this element will be associated to one of the subsystems on the right, to which it has but 1 link.

The most appropriate subsystem for this element to belong to is of course the subsystem, S, to which it is connected by 3 links. But the two-way partition algorithm is unable to assign the element x intelligently because, as it were, it has no way of seeing into the future lower levels of decomposition not yet carried out. This has actually happened, in real analysis under HIDECS 2, and has led to irritating anomalies. To avoid it, we must make use of an algorithm which surveys the entire decomposition all at once.

For any decomposition of M into disjoint sets S_1, S_2,..., define

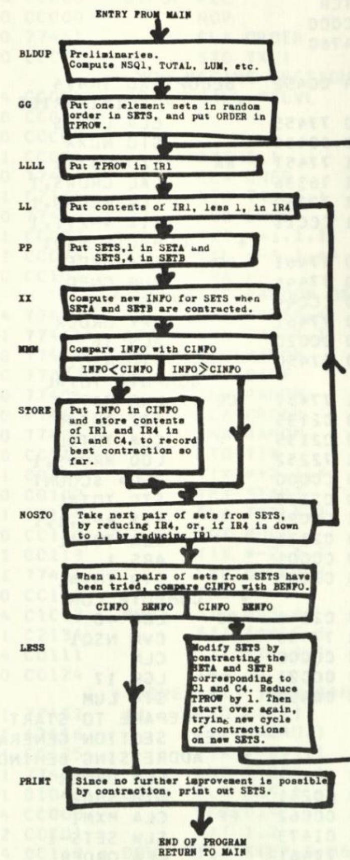
$$\text{INFO} = \frac{\left\{v - \frac{2l}{m(m-1)} \sum_{i\neq j} s_i s_j\right\}^2 \delta}{\sum_{i\neq j} s_i s_j \left(\frac{m(m-1)}{2} - \sum_{i\neq j} s_i s_j\right)}$$

where s_i is the number of elements in S_i, and δ is 1 or -1 according as the bracket is positive or negative. The task of BLDUP is to find that decomposition of M into disjoint sets, which has the minimum value of INFO.

The program is a hill-climbing program. The basic operational unit of the hill climb is the contraction of a decomposition, in which two sets of the decomposition S_i and S_j, are joined to form a single set $S_i \cup S_j$, thus giving another decomposition. The hill climb begins by placing the M one-element sets, in numerical order, in SETS. This defines the unit decomposition of M into its individual vertex sets. The program examines each possible contraction of the unit decomposition, computing INFO for each contraction, and then forms that contraction whose value of INFO is least. It then continues the same process, computing INFO for all the contractions of this new decomposition, and again forming whichever contraction is the best; the process of contraction is repeated until the program reaches a decomposition which has a lower value of INFO than any of its contractions; at this point no improvement is possible by contraction: The hill climb terminates, and the program prints out the contents of SETS.

Although it is possible that there might be decompositions, not reachable by this process of contraction, with even lower values of INFO, experiments have shown that this is not usually the case; the reason seems to be that the function INFO varies only very slowly with changes in decomposition.

BLDUP



```
*     FAP

                    00005           ENTRY   BLDUP

  TRANSFER VECTOR
00000  234664456360        COUNT
00001  476343654360        PTLVL
00002  476362256360        PTSFT

  LINKAGE DIRECTOR
00003  000000000000
00004  224324644760

00005 -0634 00 4 00452  BLDUP SXD IR4,4
                        *     PRELIMINARIES
00006  0500 00 0 77455        CLA LATIS
00007  0622 00 0 00455        STD NDXX
00010 -0534 00 1 77457  AA    LXD DAT,1
00011 -0500 00 1 76336        CAL DROWS,1
00012  0602 00 1 72252        SLW MROWS,1
00013  2 00001 1 00011        TIX *-2,1,1
                        *   COMPUTE NSQ1
00014  0500 00 0 77461  BB    CLA ORDER
00015  0402 00 0 77444        SUB CNED
00016 -0765 00 0 00044        LGR 36
00017  0200 00 0 77461        MPY ORDER
00020  0767 00 0 00020        ALS 16
00021  0622 00 0 77450        STD NSQ1
                        *   COMPUTE TOTAL
00022 -0534 00 1 77457  CC    LXD DAT,1
00023  0600 00 0 02135        STZ TOTAL
00024  0500 00 0 02135        CLA TOTAL
00025  0560 00 1 72252        LDQ MROWS,1
00026  0522 60 0 00000        XEC* $COUNT
00027  0622 00 0 02135        STD TOTAL
00030  2 00001 1 00024        TIX *-4,1,1
00031  0500 00 0 02135        CLA TOTAL
00032  0771 00 0 00001        ARS 1
00033  0622 00 0 02135        STD TOTAL
                        *   COMPUTE LUM
00034  0560 00 0 02141  DD    LDQ =0
00035  0220 00 0 77450        DVH NSQ1
00036  0760 00 0 00000        CLM
00037 -0765 00 0 00021        LGR 17
00040 -0600 00 0 00453        STQ LUM
                        *   PREPARE TO START
                        * NEXT SECTION GENERATES INDIRECT
                        *   ADDRESSING BEHIND SETS
00041  0500 00 0 77461        CLA ORDER
00042  0622 00 0 00241        STD S902
00043  0500 00 0 00062  FF    CLA MXM1
00044  0602 00 0 01473        SLW SETS-1
00045  0500 00 0 77461        CLA ORDER
00046  0622 00 0 00060        STD LOC
00047  0500 00 0 77460        CLA NWORD
00050  0400 00 0 77444        ADD CNED
00051  0771 00 0 00022        ARS 18
00052  0601 00 0 00063        STO DIFSP
00053  0774 00 1 00001        AXT 1,1
00054  0500 00 1 01474        CLA SETS,1
00055  0402 00 0 00063        SUB DIFSP
00056  1 00001 1 00057        TXI *+1,1,1
00057  0602 00 1 01474        SLW SETS,1
00060 -3 00000 1 00055  LOC   TXL *-3,1,**
00061  0020 00 0 00064        TRA *+3
00062  000000247616     MXM1  VFD 18/0,03/2,15/MACRO
00063  0 00000 0 00000  DIFSP PZE
00064  0761 00 0 00000        NOP
00065  0500 00 0 77461        CLA ORDER
00066  0622 00 0 00077        STD TXL1
                        * NOW BEGINS SECTION TO GENERATE NOS
00067  0074 00 4 00001  GG    CALL PTLVL
00070  1 00000 0 00072
00071  0 00505 0 00003
00072  0774 00 1 00001        AXT 1,1
00073  0500 00 0 77444        CLA CNED
00074  0622 00 1 02134        STD TABLE,1
00075  0400 00 0 77444        ADD CNED
00076  1 00001 1 00077        TXI *+1,1,1
00077 -3 00000 1 00074  TXL1  TXL *-3,1,**
00100  0020 00 0 00101        TRA HH
                        * GENERATE RANDOM ORDER
00101 -0534 00 4 77461  HH    LXD ORDER,4
00102 -0534 00 1 77461  START LXD ORDER,1
00103 -0500 00 0 77404  GEN   CAL RANDM
00104  0361 00 0 77372        ACL DIFF
00105  0602 00 0 77404        SLW RANDM
00106  0500 00 0 77461        CLA ORDER
00107 -0320 00 0 77404        ANA RANDM
00110  0622 00 0 00111        STD TIX
00111  2 00000 1 00113  TIX   TIX *+2,1,**
00112  0020 00 0 00102        TRA START
00113  0500 00 1 02134        CLA TABLE,1
00114 -0100 00 0 00120        TNZ MOVE
00115  2 00001 1 00113        TIX *-2,1,1
00116 -0534 00 1 77461        LXD ORDER,1
00117  0020 00 0 00113        TRA *-4
00120  0622 00 4 01062  MOVE  STD NOS,4
00121  0600 00 1 02134        STZ TABLE,1
00122  2 00001 4 00111        TIX TIX,4,1
00123  0020 00 0 00124        TRA JJ
                        * MOVE SETS INTO RANDOM PLACES
00124 -0534 00 1 77457  JJ    LXD DAT,1
00125  0600 00 1 47616        STZ MACRO,1
00126  2 00001 1 00125        TIX *-1,1,1
00127 -0534 00 1 77461        LXD ORDER,1
00130  0500 00 1 01062  LOAD  CLA NOS,1
00131 -0734 00 4 00000        PDX 0,4
00132  0774 00 2 00001        AXT 1,2
00133  2 00044 4 00140  DOWN  TIX STEP,4,36
00134 -0500 00 4 77170        CAL UNIT,4
00135  0602 60 1 01474        SLW* SETS,1
00136  2 00001 1 00130        TIX LOAD,1,1
```

```
         00137  0020 00 0 CC142          TRA KK
         00140  1 COC01 2 CC133   STEP   TXI DOWN,2,1
A        00141  CC00 CO 0 CCCC0          HTR
         00142  0500 00 0 77450   KK     CLA NSQ1
         00143  0622 CO 0 C1C67          STD SSSUM
         00144  0500 CO 0 C2135          CLA TOTAL
         00145  0622 CO 0 C1C66          STD RR
         00146  0500 00 0 C2142          CLA =C377777777777
         00147  0601 00 0 CC443          STO CINFO
         00150  0601 00 0 CC424          STO BENFO
         00151 -0534 CO 1 77461          LXD ORDER,1
         00152 -0634 CO 1 C1527   LLLL   SXD TPROW,1
         00153 -0754 00 1 CCC00   LL     PXD 0,1
         00154  0402 CO 0 77444          SUB CNED
         00155 -0734 CO 4 CCC00          PDX C,4
         00156 -0534 CO 2 77460   PP     LXD NWORD,2
         00157 -0500 60 1 C1474          CAL* SETS,1
         00160  C602 CO 2 01507          SLW SETA,2
         00161  2 00001 2 CC157          TIX *-2,2,1
         00162 -0534 00 2 77460          LXD NWORD,2
         00163 -0500 60 4 C1474          CAL* SETS,4
         00164  C602 CO 2 C1522          SLW SETB,2
         0813  2 8801 2 C0163         TIX *-2Z ,1
         00166  0020 CO 0 C0167          TRA XX
                                  * NEXT SECTION COMPUTES INFO
         00167 -0634 00 I C2137   XX     SXD XY1,1
         00170 -0634 00 4 C214C          SXD XY4,4
         00171  0600 CO C C1523          STZ SA
         00172 -0534 00 2 7746C          LXD NWORD,2
         00173  0500 00 0 C1523          CLA SA
         00174  0560 CO 2 01507          LDQ SETA,2
         00175  0522 60 0 CCCC0          XEC* $COUNT
         00176  0622 00 0 C1523          STD SA
         00177  2 C0001 2 CC173          TIX *-4,2,1
         002C0  0600 CO C C1524          STZ SB
         00201 -0534 CO 2 77460          LXD NWORD,2
         00202  0500 00 0 01524          CLA SB
         00203  0560 CO 2 C1522          LDQ SETB,2
         00204  0522 60 C CCC0C          XEC* $COUNT
         00205  0622 00 0 C1524          STD SB
         00206  2 00001 2 00202          TIX *-4,2,1
         00207  0500 00 C C1523          CLA SA
         00210 -0765 00 0 CCC44          LGR 36
         00211  0200 00 0 01524          MPY SB
         00212  0767 00 0 CC021          ALS 17
         00213  0622 00 0 C1525          STD SASB
         00214  0600 CO 0 C0423          STZ ARB
         00215  0774 CO 4 CCC01          AXT 1,4
         00216  0774 00 I CCC01          AXT 1,1
         00217  0560 00 4 C1507   LDTST  LDQ SETA,4
         00220 -0634 CO 4 C1526          SXD SXRD,4
         00221  0774 CO 4 CCC01          AXT 1,4
         00222  0162 00 0 C0236   TQPP   TQP SKIP
         00223 -0600 C0 0 C2136          STQ WAIT
         00224 -0534 00 2 77460   AROW   LXD NWORD,2
         00225 -0500 CO 2 C1522          CAL SETB,2
```

```
00226 -0320 60 1 76742          ANA* MATAX,1
00227  0100 CO 0 C0234          TZE *+5
00230 -0765 00 0 CC044          LGR 36
00231  0500 00 0 CC423          CLA ARB
00232  0522 60 0 CCCC0          XEC* $COUNT
00233  0622 00 0 C0423          STD ARB
00234  2 00001 2 C0225          TIX AROW+1,2,1
00235  0560 00 C C2136          LDQ WAIT
00236 -0773 CO C CCC01   SKIP   RQL 1
00237  1 00001 1 C024C          TXI *+1,1,1
00240  1 0000C1 4 C0241         TXI *+1,4,1
00241  3 0CC00 1 CC245   S902   TXH RRRR,1,**
00242 -3 C0044 4 CC222          TXL TQPP,4,36
00243 -0534 CO 4 01526          LXD SXRD,4
00244  1 C0001 4 C0217          TXI LDTST,4,1
00245  C020 00 C CC246   RRRR   TRA YY
                         * YY ACTUALLY DOES THE DIVISION
00246  0500 CO 0 01066   YY     CLA RR
00247  0402 00 0 C0423          SUB ARB
00250  0622 00 C 01C63          STD NRR
00251  0500 CO 0 C1C67          CLA SSSUM
00252  04C2 CO 0 C1525          SUB SASB
00253  0622 00 0 C1C64          STD NSSUM
00254  0500 00 C 7745C          CLA NSQ1
00255  0402 CO C C1C64          SUB NSSUM
00256  0765 C0 0 CC043          LRS 35
00257  0200 00 0 C1C64          MPY NSSUM
00260  0771 C0 0 CCC01          ARS 1
00261  C601 00 0 CC425          STO BOTT
00262 -0100 00 0 C0265          TNZ *+3
00263  0600 C0 0 C0451          STZ INFO
00264  0020 CO 0 C0303          TRA TAGET
00265  0500 CO 0 C1C64          CLA NSSUM
00266  0765 00 C CC043          LRS 35
00267  0200 00 0 C0453          MPY LUM
00270  0763 00 C CCC21          LLS 17
00271  04C2 CO 0 C1C63   SAD    SUB NRR
00272  0760 CO 0 CCC02          CHS
00273  0601 00 0 00454          STO MULT
00274  0120 CO 0 C0276          TPL *+2
00275  0760 00 0 CCC02          CHS
00276  0765 00 0 CC043          LRS 35
00277  0200 00 0 C0454          MPY MULT
00300  0765 00 C CCC01          LRS 1
00301  0220 00 0 CC425          DVH BOTT
00302 -0600 CO C C0451          STQ INFO
00303 -0534 00 1 C2137   TAGET  LXD XY1,1
00304 -0534 CO 4 C2140          LXD XY4,4
00305  C020 CO 0 C0306          TRA MMM
                         * END OF SECTION WHICH COMPUTES INFO
                         * NEXT SECTION COMPARES INFO WITH CINFO
00306  0500 00 0 C0451   MMM    CLA INFO
00307  0340 CO 0 CC443          CAS CINFO
00310  0020 00 0 C0330          TRA NOSTO
00311  0020 00 0 00330          TRA NOSTO
00312  0020 00 0 00313          TRA STORE
```

```
00313 -0634 00 4 C0426   STORE SXD C4,4
00314 -0634 CO I C0427         SXD C1,1
00315 -0534 CO 2 7746C         LXD NWORD,2
00316 -0500 00 2 01507         CAL SETA,2
00317 -0501 00 2 01522         ORA SETB,2
00320  0602 00 2 00442         SLW CET,2
00321  2 00001 2 C0316         TIX *-3,2,1
00322  0500 00 0 01064         CLA NSSUM
00323  0601 00 0 C0444         STO CSSUM
00324  0500 C0 0 C1063         CLA NRR
00325  0601 00 0 C0445         STO CRR
00326  0500 00 0 00451         CLA INFO
00327  0601 00 0 C0443         STO CINFO
00330  2 00001 4 C0156   NOSTO TIX PP,4,1
00331 -3 00002 I C0333         TXL *+2,1,2
00332  2 00001 I 00153         TIX LL,1,1
00333  0500 00 0 C0443         CLA CINFO
00334  0340 00 0 C0424         CAS BENFO
00335  0020 C0 0 C0366         TRA PRINT
00336  C020 00 0 00366         TRA PRINT
00337  0020 00 0 C0340         TRA LESS
00340 -0534 00 4 C0426   LESS  LXD C4,4
00341 -0534 C0 I C0427         LXD C1,1
00342 -0534 00 2 77460         LXD NWORD,2
00343 -0500 00 2 00442         CAL CET,2
00344  0602 60 4 01474         SLW* SETS,4
00345  2 C0001 2 C0343         TIX *-2,2,1
00346  0500 00 0 C0444         CLA CSSUM
00347  0601 00 0 C1067         STO SSSUM
00350  0500 00 0 00445         CLA CRR
00351  0601 C0 0 01066         STO RR
00352  0500 00 0 00443         CLA CINFO
00353  0601 00 0 C0424         STO BENFO
00354 -0534 00 4 01527         LXD TPROW,4
00355 -0534 00 2 77460         LXD NWORD,2
00356 -0500 60 4 01474         CAL* SETS,4
00357  0602 60 I C1474         SLW* SETS,1
00360  0600 60 4 01474         STZ* SETS,4
00361  2 00001 2 C0356         TIX *-3,2,1
00362 -0534 00 1 01527         LXD TPROW,1
00363 -3 00002 1 C0365         TXL *+2,1,2
00364  2 00001 I C0152         TIX LLLL,1,1
00365  0020 00 0 C0420         TRA OUT
00366  0500 00 0 77460   PRINT CLA NWORD
00367  0622 00 0 C0446         STD HOLD
00370 -0534 00 2 77457         LXD DAT,2
00371 -0500 C0 2 47616         CAL MACRO,2
00372  0602 00 2 66166         SLW INMAT,2
00373  2 00001 2 00371         TIX *-2,2,1
00374 -0534 00 4 C0446   FRESH LXD HOLD,4
00375 -0534 00 2 77460         LXD NWORD,2
00376 -0500 00 4 66166         CAL INMAT,4
00377  0602 00 2 77416         SLW SET,2
00400  2 00001 4 00401         TIX *+1,4,1
00401  2 00001 2 C0376         TIX *-3,2,1
00402  0500 00 0 00446         CLA HOLD

00403  0400 00 0 77460         ADD NWORD
00404  0400 00 0 77444         ADD CNED
00405  0622 00 0 CC446         STD HOLD
00406 -0534 00 2 77460         LXC NWORD,2
00407  0760 00 0 C0000         CLM
00410 -0501 00 2 77416         ORA SET,2
00411  2 00001 2 C0410         TIX *-1,2,1
00412  0100 C0 0 CC417         TZE ENDPT
00413  0074 C0 4 CC002         CALL PTSET
00414  1 00000 0 C0416
00415  0 20607 C CCC03
00416  0020 00 0 C0374         TRA FRESH
00417  C020 00 0 00420   ENDPT TRA OUT
00420 -0534 00 4 CC452   OUT   LXD IR4,4
00421  0020 C0 4 CCC01         TRA 1,4
00422  0 COOCO 0 CCC00         PZE
00423  0 CCC00 0 CCC00   ARB
00424  0 00000 0 CCC00   BENFO
00425  0 00C00 C CCC00   BOTT
00426  0 CCC00 0 CCC00   C4
00427  0 CCC00 0 CCC00   C1
00442                          BES 10
00442  0 00000 C CCC00   CET
00443  0 CCOCO C CCC00   CINFO
00444  0 CCC00 0 CCCC0   CSSUM
00445  0 00000 0 CCC00   CRR
00446  0 00000 0 CCC00   HOLD
00447  0 COCCO 0 CCC00   INOPN
00450  0 C0000 0 CCC00   INBEN
00451  0 00000 0 C0000   INFO
00452  0 00C00 C CCC00   IR4
00453  0 000C0 0 CCC00   LUM
C0454  0 CCCC0 0 CCC00   MULT
00455  0 00000 0 CCC00   NDXX
01062                          BES 260
01062  0 000C0 0 CCCC0   NOS
01063  0 00000 0 CCC00   NRR
01064  0 00000 0 CCC00   NSSUM
01065  0 C00C0 0 CCC00   OPNFO
01066  0 COCC0 0 CCC00   RR
01067  0 CCC00 0 CCC00   SSSUM
01474                          BES 260
01474  0 00000 C CCC00   SETS
01507                          BES 10
01507  0 00000 0 CC000   SETA
01522                          BES 10
01522  0 00C00 0 CC000   SETB
01523  0 00CC0 0 CCC00   SA
01524  0 00000 0 CCC00   SB
01525  0 00000 0 CCOC0   SASB
01526  0 00000 C CCC00   SXRD
01527  0 000C0 0 CCC00   TPROW
02134                          BES 260
02134  0 000C0 0 CC000   TABLE
02135  0 00CC0 0 CCC00   TOTAL
02136  0 000C0 0 CCC00   WAIT
```

```
02137  0 00C00 0 00C00    XY1
02140  0 000C0 0 CC000    XY4
                      *   COMMCN BLOCK FROM HIDECS 2
          77462               COMMCN -1
          77462        INDIC  COMMCN 1
          77461        ORDER  COMMCN 1
          77460        NWORD  COMMCN 1
          77457        DAT    COMMCN 1
          77456        LGTH   COMMCN 1
          77455        LATIS  COMMCN 1
          77454        NBITH  COMMCN 1
          77453        NBITL  COMMCN 1
          77452        NBIT1  COMMCN 1
          77451        NBIT   COMMCN 1
          77450        NSQ1   COMMCN 1
          77447        OPRMN  COMMCN 1
          77446        ATOMO  COMMCN 1
          77445        ATOM   COMMCN 1
          77444        ONED   COMMCN 1
          77443        D36    COMMCN 1
          77442        ATCOX  COMMCN 10
          77430        ATOX   COMMON 10
          77416        SET    COMMCN 10
          77404        RANCM  COMMCN 1C
          77372        DIFF   COMMCN 10
          77360        CONVT  COMMON 40
          77310        DATA   COMMCN 40
          7724C        MATA   COMMCN 40
          77170        UNIT   COMMCN 40
          77120        COMUN  COMMCN 40
          77C50        EQLS   COMMCN 20
          77C24        SECTS  COMMCN 50
          76742        MATAX  COMMCN 260
          76336        DROWS  COMMCN 2100
          72252        MROWS  COMMCN 2100
          66166        INMAT  COMMCN 5400
          53536        ATMS   COMMCN 2C00
          47616        MACRO  COMMCN 7000
                              END

  LITERALS
02141   00000CCC0C000
02142   377777777777
```

The decomposition of the sample graph follows:

```
 2  3
14 15 16
 4  5  6
12 13
 1 17
 7  8  9 10 11
```


STABL

This program is very similar to BLDUP. For any decomposition of M into disjoint sets $S_1, S_2, \ldots$, define the function EXP as:

$$\text{EXP} = \frac{\left\{\sum_i \left[l_i \cdot \frac{m(m-1)}{2l} - \frac{s_i(s_i-1)+1}{2}\right]\right\}^2 \delta}{\sum_i \left(\frac{s_i(s_i-1)+1}{2}\right) 2^{-2s_i}}$$

, where δ is +1 or -1 according as the top bracket is positive or negative.

The task of STABL is to find that decomposition of M into disjoint sets, for which EXP is maximum. It has been shown elsewhere that the decomposition so obtained is maximally stable under the addition of new and unknown elements to the system M.*

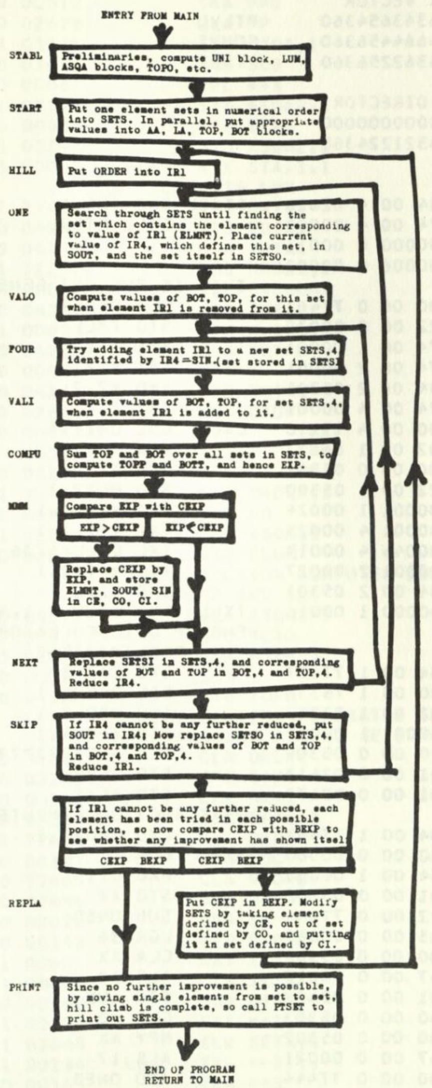
This function EXP varies sharply even over slight variation in the decomposition. The crude hill-climb by successive contractions, used in BLDUP, is therefore unsuitable for STABL. In fact, in experience, even for small and simple systems, a hill-climb based on contraction failed to find the decomposition with the best value of EXP. Instead STABL is based on the following procedure.

Start with the unit decomposition in SETS, as before. The basic operation consists of moving one element, out of the set it happens to be in, and adding it to each of the other sets in turn, computing EXP for each new decomposition so obtained. This is done for each element. The best of

---

*Actually EXP differs slightly from the criterion function given in (1963 b); the changes make it more continuous in the search space; the original function had such severe discontinuities that the hill climb would not work at all. l_i is the number of links in the subsystem S_i.

all the decompositions so obtained is thus the best decomposition to be obtained by moving a single element. The program makes this change; and then begins the cycle again. The program terminates when it finds a decomposition whose value of EXP is higher than that for any decomposition obtainable from it by moving a single element.



```
*      FAP

                     00005          ENTRY    STABL

  TRANSFER VECTOR
00000  476343654360        PTLVL
00001  234664456360        COUNT
00002  476362256360        PTSET

  LINKAGE DIRECTOR
00003  000000000000
00004  626321224360


00005 -0634 00 4 02626   STABL SXD IR4,4
00006  0074 00 4 00000         CALL PTLVL
00007  1 00000 0 00011
00010  0 00006 0 00003
                         *THIS IS THE UNI GENERATOR
00011  0500 00 0 77461         CLA ORDER
00012  0622 00 0 00030         STD TXLL
00013  0774 00 1 00001         AXT 1,1
00014  0774 00 2 00001         AXT 1,2
00015 -0634 00 2 05301         SXD X2,2
00016  0774 00 4 00001         AXT 1,4
00017 -0500 00 4 77170   CALU  CAL UNIT,4
00020  0602 00 1 04673         SLW UNI,1
00021  0500 00 0 05301         CLA X2
00022  0622 00 1 05300         STD UNI2,1
00023  1 00001 1 00024         TXI *+1,1,1
00024  1 00001 4 00025         TXI *+1,4,1
00025 -3 00044 4 00017         TXL CALU,4,36
00026  1 00001 2 00027         TXI *+1,2,1
00027 -0634 00 2 05301         SXD X2,2
00030 -3 00000 1 00016   TXLL  TXL CALU-1,1,**
                         *END OF UNI GENERATOR
                         *      PRELIMINARIES
00031 -0534 00 1 77457         LXD DAT,1
00032 -0500 00 1 76336         CAL DROWS,1
00033  0602 00 1 72252         SLW MROWS,1
00034  2 00001 1 00032         TIX *-2,1,1
00035  0500 00 0 05306         CLA =O777777777777
00036  0601 00 0 02613         STO CEXP
00037  0601 00 0 02177         STO BEXP
                         *NEXT SECTION COMPUTES ASQA BLOCKS
00040 -0534 00 1 77461         LXD ORDER,1
00041  0760 00 0 00000   STA   CLM
00042 -0754 00 1 00000         PXD 0,1
00043  0601 00 0 05302         STO XX
00044  0402 00 0 77444         SUB ONED
00045 -0765 00 0 00044         LGR 36
00046  0500 00 0 05302         CLA XX
00047  0767 00 0 00012         ALS 10
00050  0601 00 0 03652         STO SUB
00051  0500 00 0 05303         CLA =0
00052  0200 00 0 05302         MPY XX
00053  0767 00 0 00021         ALS 17
00054  0400 00 0 77444         ADD ONED
00055  0771 00 0 00001         ARS 1
```

```
00056  0601 00 0 02617         STO DTOFP
00057 -0625 00 0 00542         STL TRAP
00060  0020 00 0 00512         TRA FPG
00061  0500 00 0 02623         CLA FP
00062  0601 00 1 01571         STO ASQA,1
00063  0402 00 0 03652         SUB SUB
00064  0120 00 0 00067         TPL *+3
00065  0600 00 1 02176         STZ ASQA2,1
00066  0020 00 0 00070         TRA TIXS
00067  0601 00 1 02176         STO ASQA2,1
00070  2 00001 1 00041   TIXS  TIX STA,1,1
                         *   COMPUTE LUM
00071 -0534 00 1 77457   CC    LXD DAT,1
00072  0600 00 0 04266         STZ TOTAL
00073  0500 00 0 04266         CLA TOTAL
00074  0560 00 1 72252         LDQ MROWS,1
00075  0522 60 0 00001         XEC* $COUNT
00076  0622 00 0 04266         STD TOTAL
00077  2 00001 1 00073         TIX *-4,1,1
00100  0500 00 0 04266         CLA TOTAL
00101  0771 00 0 00001         ARS 1
00102  0622 00 0 04266         STD TOTAL
00103  0601 00 0 02617         STO DTOFP
00104 -0625 00 0 00542         STL     TRAP
00105  0020 00 0 00512         TRA     FPG
00106  0500 00 0 02623         CLA     FP
00107 -0534 00 1 77461         LXD ORDER,1
00110  0560 00 0 05303         LDQ =0
00111  0240 00 1 01571         FDH ASQA,1
00112 -0600 00 0 02622         STQ FLUM
                         * NEXT SECTION COMPUTES ZERO VALUES
                         * FOR TOP AND BOT
00113  0500 00 0 02175         CLA ASQA2-1
00114  0601 00 0 02611         STO BOTO
00115  0500 00 0 01570         CLA ASQA-1
00116 -0760 00 0 00003         SSM
00117  0601 00 0 04264         STO TOPO
                         * NEXT SECTION GENERATES INDIRECT
                         * ADDRESSING BEHIND SETS AS IN GENER
00120  0500 00 0 77461         CLA ORDER
00121  0622 00 0 03240         STD S902
00122  0500 00 0 00141   FF    CLA MXM1
00123  0602 00 0 03644         SLW SETS-1
00124  0500 00 0 77461         CLA ORDER
00125  0622 00 0 00137         STD LOC
00126  0500 00 0 77460         CLA NWORD
00127  0400 00 0 77444         ADD ONED
00130  0771 00 0 00022         ARS 18
00131  0601 00 0 00142         STO DIFSP
00132  0774 00 1 00001         AXT 1,1
00133  0500 00 1 03645         CLA SETS,1
00134  0402 00 0 00142         SUB DIFSP
00135  1 00001 1 00136         TXI *+1,1,1
00136  0602 00 1 03645         SLW SETS,1
00137 -3 00000 1 00134   LOC   TXL *-3,1,**
00140  0020 00 0 00143         TRA *+3
00141  000000247616      MXM1  VFD 18/0,03/2,15/MACRO
00142  0 00000 0 00000   DIFSP PZE
```

```
00143  0761 00 0 00000          NOP
                         * NEXT SECTION PUTS ONE ELEMENT SETS
                         *    IN NUMERICAL ORDER BEHIND SETS
                         *    AND SETS ONE ELEMENT VALUES FOR
                         *    BOT, TOP, AA AND LA
00144 -0534 00 1 77461    START LXD ORDER,1
00145  0500 00 1 05300          CLA UNI2,1
00146 -0734 00 2 00000          PDX 0,2
00147 -0500 00 1 04673          CAL UNI,1
00150  0602 60 1 03645          SLW* SETS,1
00151  2 00001 1 00145          TIX *-4,1,1
00152 -0534 00 1 77461          LXD ORDER,1
00153  0500 00 0 77444          CLA ONED
00154  0622 00 1 01164          STD AA,1
00155  0600 00 1 03233          STZ LA,1
00156  2 00001 1 00154          TIX *-2,1,1
00157 -0534 00 1 77461          LXD ORDER,1
00160  0500 00 0 04264          CLA TOPO
00161  0601 00 1 04263          STO TOP,1
00162  2 00001 1 00161          TIX *-1,1,1
00163 -0534 00 1 77461          LXD ORDER,1
00164  0500 00 0 02611          CLA BOTO
00165  0601 00 1 02610          STO BOT,1
00166  2 00001 1 00165          TIX *-1,1,1
                         *NOW START HILL CLIMB
                         * NEXT SECTION PICKS OUT SET WHICH
                         *    HAS NEXT ELEMENT IN IT
00167 -0534 00 1 77461    HILL  LXD ORDER,1
00170 -0534 00 4 77461    ONE   LXD ORDER,4
00171  0500 00 1 05300          CLA UNI2,1
00172 -0734 00 2 00000          PDX 0,2
00173 -0500 00 1 04673          CAL UNI,1
00174 -0320 60 4 03645          ANA* SETS,4
00175 -0100 00 0 00200          TNZ *+3
00176  2 00001 4 00173          TIX *-3,4,1
00177  0000 00 0 00000          HTR
00200  0560 60 4 03645          LDQ* SETS,4
00201 -0600 00 0 03647          STQ SETSO
00202 -0634 00 1 02621          SXD ELMNT,1
00203  0322 60 4 03645          ERA* SETS,4
00204  0602 60 4 03645          SLW* SETS,4
00205  0500 00 4 02610          CLA BOT,4
00206  0601 00 0 02203          STO BOUT
00207  0500 00 4 04263          CLA TOP,4
00210  0601 00 0 03656          STO TOUT
00211 -0634 00 4 03651          SXD SOUT,4
                         *NEXT SECTION DEFINES NEW BOT FOR SOUT
00212  0500 00 4 01164    VALO  CLA AA,4
00213  0402 00 0 77444          SUB ONED
00214 -0734 00 4 00000          PDX 0,4
00215  0601 00 0 03652          STO SUB
00216  0500 00 4 02176          CLA ASQA2,4
00217 -0534 00 4 03651          LXD SOUT,4
00220  0601 00 4 02610          STO BOT,4
                         *NEXT SECTION DEFINES NEW TOP FOR SOUT
00221 -0534 00 2 77460          LXD NWORD,2
00222  0600 00 0 03237          STZ RL
00223 -0500 60 1 76742    OCNT  CAL* MATAX,1
```

```
00224 -0320 60 4 03645          ANA* SETS,4
00225 -0765 00 0 00044          LGR 36
00226  0500 00 0 03237          CLA RL
00227  0522 60 0 00001          XEC* $COUNT
00230  0622 00 0 03237          STD RL
00231  2 00001 2 00223          TIX OCNT,2,1
00232  0500 00 4 03233          CLA LA,4
00233 -0634 00 4 03651          SXD SOUT,4
00234 -0534 00 4 03652          LXD SUB,4
00235  0402 00 0 03237          SUB RL
00236  0601 00 0 02617          STO DTOFP
00237 -0625 00 0 00542          STL TRAP
00240  0020 00 0 00512          TRA FPG
00241  0500 00 0 02623          CLA FP
00242  0240 00 0 02622          FDH FLUM
00243  0131 00 0 00000          XCA
00244  0302 00 4 01571          FSB ASQA,4
00245 -0534 00 4 03651          LXD SOUT,4
00246  0601 00 4 04263          STO TOP,4
                         * NEXT SECTION TRIES ADDING CHOSEN
                         *    ELEMENT TO VARIOUS NEW SETS
00247 -0534 00 4 77461          LXD ORDER,4
00250 -0634 00 4 03650    FOUR  SXD SIN,4
00251  0500 00 0 03651          CLA SOUT
00252  0340 00 0 03650          CAS SIN
00253  0020 00 0 00256          TRA *+3
00254  0020 00 0 00406          TRA SKIP
00255  0020 00 0 00256          TRA *+1
00256  0500 00 4 02610          CLA BOT,4
00257  0601 00 0 02201          STO BIN
00260  0500 00 4 04263          CLA TOP,4
00261  0601 00 0 03654          STO TIN
00262 -0634 00 4 03650          SXD SIN,4
                         *NEXT SECTION DEFINES NEW BOT FOR SIN
00263  0500 00 4 01164    VALI  CLA AA,4
00264  0400 00 0 77444          ADD ONED
00265 -0734 00 4 00000          PDX 0,4
00266  0601 00 0 03652          STO SUB
00267  0500 00 4 02176          CLA ASQA2,4
00270 -0534 00 4 03650          LXD SIN,4
00271  0601 00 4 02610          STO BOT,4
                         *NEXT SECTION DEFINES NEW TOP FOR SIN
00272 -0534 00 2 77460          LXD NWORD,2
00273  0600 00 0 03237          STZ RL
00274 -0500 60 1 76742    ICNT  CAL* MATAX,1
00275 -0320 60 4 03645          ANA* SETS,4
00276 -0765 00 0 00044          LGR 36
00277  0500 00 0 03237          CLA RL
00300  0522 60 0 00001          XEC* $COUNT
00301  0622 00 0 03237          STD RL
00302  2 00001 2 00274          TIX ICNT,2,1
00303  0500 00 4 03233          CLA LA,4
00304 -0634 00 4 03650          SXD SIN,4
00305 -0534 00 4 03652          LXD SUB,4
00306  0400 00 0 03237          ADD RL
00307  0601 00 0 02617          STO DTOFP
00310 -0625 00 0 00542          STL TRAP
00311  0020 00 0 00512          TRA FPG
```

```
00312  0500 00 0 02623         CLA FP
00313  0240 00 0 02622         FDH FLUM
00314  0131 00 0 00000         XCA
00315  0302 00 4 01571         FSB ASQA,4
00316 -0534 00 4 03650         LXD SIN,4
00317  0601 00 4 04263         STO TOP,4
00320  0500 00 1 05300         CLA UNI2,1
00321 -0734 00 2 00000         PDX 0,2
00322  0560 60 4 03645         LDQ* SETS,4
00323 -0600 00 0 03646         STQ SETSI
00324 -0500 00 1 04673         CAL UNI,1
00325 -0602 60 4 03645         ORS* SETS,4
                         *NEXT SECTION COMPUTES VALUE OF EXP
00326 -0534 00 1 77461   COMPU LXD ORDER,1
00327  0600 00 0 04265         STZ TOPP
00330  0600 00 0 02612         STZ BOTT
00331  0500 00 0 04265         CLA TOPP
00332  0300 00 1 04263         FAD TOP,1
00333  0601 00 0 04265         STO TOPP
00334  0500 00 0 02612         CLA BOTT
00335  0300 00 1 02610         FAD BOT,1
00336  0601 00 0 02612         STO BOTT
00337  2 00001 1 00331         TIX *-6,1,1
00340 -0534 00 1 02621         LXD ELMNT,1
00341  0500 00 0 04265         CLA TOPP
00342  0120 00 0 00344         TPL *+2
00343  0760 00 0 00002         CHS
00344  0131 00 0 00000         XCA
00345  0260 00 0 04265         FMP TOPP
00346  0560 00 0 05303         LDQ =0
00347  0240 00 0 02612         FDH BOTT
00350 -0600 00 0 02620         STQ EXP
                         * NEXT SECTION COMPARES EXP WITH CEXP
                         *   AND REPLACES IT IF BETTER
00351  0500 00 0 02620   MMM   CLA EXP
00352  0340 00 0 02613         CAS CEXP
00353  0020 00 0 00356         TRA *+3
00354  0020 00 0 00377         TRA NEXT
00355  0020 00 0 00377         TRA NEXT
00356  0601 00 0 02613         STO CEXP
00357  0500 00 0 02621         CLA ELMNT
00360  0601 00 0 02614         STO CE
00361  0500 00 0 03651         CLA SOUT
00362  0601 00 0 02616         STO CO
00363  0500 00 0 03650         CLA SIN
00364  0601 00 0 02615         STO CI
00365 -0534 00 4 03650         LXD SIN,4
00366  0500 00 4 04263         CLA TOP,4
00367  0601 00 0 03653         STO TI
00370  0500 00 4 02610         CLA BOT,4
00371  0601 00 0 02200         STO BI
00372 -0534 00 4 03651         LXD SOUT,4
00373  0500 00 4 04263         CLA TOP,4
00374  0601 00 0 03655         STO TO
00375  0500 00 4 02610         CLA BOT,4
00376  0601 00 0 02202         STO BO
                         * NEXT SECTION PUTS SETS BACK TO NORMAL
                         *    TO TRY NEXT RELOCATION OF ELEMENT
00377 -0534 00 4 03650   NEXT  LXD SIN,4
00400 -0500 00 0 03646         CAL SETSI
00401  0602 60 4 03645         SLW* SETS,4
00402  0500 00 0 03654         CLA TIN
00403  0601 00 4 04263         STO TOP,4
00404  0500 00 0 02201         CLA BIN
00405  0601 00 4 02610         STO BOT,4
00406  2 00001 4 00250   SKIP  TIX FOUR,4,1
00407 -0534 00 4 03651         LXD SOUT,4
00410 -0500 00 0 03647         CAL SETSO
00411  0602 60 4 03645         SLW* SETS,4
00412  0500 00 0 03656         CLA TOUT
00413  0601 00 4 04263         STO TOP,4
00414  0500 00 0 02203         CLA BOUT
00415  0601 00 4 02610         STO BOT,4
00416  2 00001 1 00170         TIX ONE,1,1
                         * NEXT SECTION TESTS TO SEE WHETHER
                         *   THE LAST CYCLE OF THE HILL CLIMB
                         *   HAS IMPROVED THE DECOMPOSITION,
                         *   AND REPLACES IF THE ANSWER IS YES
00417  0500 00 0 02177         CLA BEXP
00420  0340 00 0 02613         CAS CEXP
00421  0000 00 0 00000         HTR
00422  0020 00 0 00543         TRA PRINT
00423  0500 00 0 02613   REPLA CLA CEXP
00424  0601 00 0 02177         STO BEXP
00425 -0534 00 1 02614         LXD CE,1
00426 -0534 00 4 02616         LXD CO,4
00427  0500 00 1 05300         CLA UNI2,1
00430 -0734 00 2 00000         PDX 0,2
00431 -0500 00 1 04673         CAL UNI,1
00432  0322 60 4 03645         ERA* SETS,4
00433  0602 60 4 03645         SLW* SETS,4
00434  0500 00 4 01164         CLA AA,4
00435  0402 00 0 77444         SUB ONED
00436  0601 00 4 01164         STO AA,4
00437 -0534 00 2 77460         LXD NWORD,2
00440  0600 00 0 03237         STZ RL
00441 -0500 60 1 76742         CAL* MATAX,1
00442 -0320 60 4 03645         ANA* SETS,4
00443 -0765 00 0 00044         LGR 36
00444  0500 00 0 03237         CLA RL
00445  0522 60 0 00001         XEC* $COUNT
00446  0622 00 0 03237         STD RL
00447  2 00001 2 00441         TIX *-6,2,1
00450  0500 00 4 03233         CLA LA,4
00451  0402 00 0 03237         SUB RL
00452  0601 00 4 03233         STO LA,4
00453  0500 00 0 03655         CLA TO
00454  0601 00 4 04263         STO TOP,4
00455  0500 00 0 02202         CLA BO
00456  0601 00 4 02610         STO BOT,4
00457 -0534 00 4 02615         LXD CI,4
00460  0500 00 1 05300         CLA UNI2,1
00461 -0734 00 2 00000         PDX 0,2
00462 -0500 00 1 04673         CAL UNI,1
00463 -0602 60 4 03645         ORS* SETS,4
00464  0500 00 4 01164         CLA AA,4
```

```
00465  0400 00 0 77444          ADD ONED
00466  0601 00 4 01164          STO AA,4
00467 -0534 00 2 77460          LXD NWORD,2
00470  0600 00 0 03237          STZ RL
00471 -0500 60 1 76742          CAL* MATAX,1
00472 -0320 60 4 03645          ANA* SETS,4
00473 -0765 00 0 00044          LGR 36
00474  0500 00 0 03237          CLA RL
00475  0522 60 0 00001          XEC* $COUNT
00476  0622 00 0 03237          STD RL
00477  2 00001 2 00471          TIX *-6,2,1
00500  0500 00 4 03233          CLA LA,4
00501  0400 00 0 03237          ADD RL
00502  0601 00 4 03233          STO LA,4
00503  0500 00 0 03653          CLA TI
00504  0601 00 4 04263          STO TOP,4
00505  0500 00 0 02200          CLA BI
00506  0601 00 4 02610          STO BOT,4
00507  0020 00 0 00167          TRA HILL
00510 -0534 00 4 02626   OUT    LXD IR4,4
00511  0020 00 4 00001          TRA 1,4
                        * SUBROUTINE FOR GENERATING FLOATING
                        *  POINT VERSION OF DECREMENT INTEGERS
00512 -0500 00 0 00542   FPG    CAL      TRAP
00513  0400 00 0 05304          ADD      =1
00514  0602 00 0 00542          SLW      TRAP
00515 -0634 00 2 03236          SXD      R2,2
00516  0774 00 2 00001          AXT      1,2
00517  0500 00 0 02617          CLA      DTOFP
00520 -0100 00 0 00523          TNZ ALS
00521  0600 00 0 02623          STZ FP
00522  0020 00 0 00541          TRA TRAP-1
00523  0767 00 0 00001   ALS    ALS      1
00524 -0760 00 0 00001          PBT
00525  1 00001 2 00523          TXI      ALS,2,1
00526  0771 00 0 00011          ARS      9
00527  0601 00 0 02623          STO      FP
00530  0754 00 2 00000          PXA      0,2
00531  0402 00 0 05305          SUB      =146
00532  0767 00 0 00033          ALS      27
00533  0760 00 0 00003          SSP
00534  0400 00 0 02623          ADD      FP
00535  0601 00 0 02623          STO      FP
00536  0760 00 0 00000          CLM
00537  0300 00 0 02623          FAD FP
00540  0601 00 0 02623          STO FP
00541 -0534 00 2 03236          LXD      R2,2
00542  0020 00 0 00000   TRAP   TRA      **
                        * NEXT SECTION PRINTS OUT COMPLETE
                        *    DECOMPOSITION AT END OF HILL CLIMB
00543 -0534 00 1 77461   PRINT  LXD ORDER,1
00544 -0534 00 2 77460          LXD NWORD,2
00545 -0500 60 1 03645          CAL* SETS,1
00546  0602 00 2 77416          SLW SET,2
00547  2 00001 2 00545          TIX *-2,2,1
00550 -0634 00 1 02625          SXD IR1,1
00551  0074 00 4 00002          CALL PTSET
00552  1 00000 0 00554
```

```
00553  0 01047 0 00003
00554 -0534 00 1 02625                 LXD IR1,1
00555  2 00001 1 00544                 TIX PRINT+1,1,1
00556  0020 00 0 00510                 TRA OUT
00557  0 00000 0 00000                 PZE
01164                                  BES 260
01164  0 00000 0 00000   AA
01571                                  BES 260
01571  0 00000 0 00000   ASQA
02176                                  BES 260
02176  0 00000 0 00000   ASQA2
02177  0 00000 0 00000   BEXP
02200  0 00000 0 00000   BI
02201  0 00000 0 00000   BIN
02202  0 00000 0 00000   BO
02203  0 00000 0 00000   BOUT
02610                                  BES 260
02610  0 00000 0 00000   BOT
02611  0 00000 0 00000   BOTO
02612  0 00000 0 00000   BOTT
02613  0 00000 0 00000   CEXP
02614  0 00000 0 00000   CE
02615  0 00000 0 00000   CI
02616  0 00000 0 00000   CO
02617  0 00000 0 00000   DTOFP
02620  0 00000 0 00000   EXP
02621  0 00000 0 00000   ELMNT
02622  0 00000 0 00000   FLUM
02623  0 00000 0 00000   FP
02624  0 00000 0 00000   HOLD
02625  0 00000 0 00000   IR1
02626  0 00000 0 00000   IR4
03233                                  BES 260
03233  0 00000 0 00000   LA
03234  0 00000 0 00000   LUM
03235  0 00000 0 00000   RI
03236  0 00000 0 00000   R2
03237  0 00000 0 00000   RL
03240  0 00000 0 00000   S902
03645                                  BES 260
03645  0 00000 0 00000   SETS
03646  0 00000 0 00000   SETSI
03647  0 00000 0 00000   SETSO
03650  0 00000 0 00000   SIN
03651  0 00000 0 00000   SOUT
03652  0 00000 0 00000   SUB
03653  0 00000 0 00000   TI
03654  0 00000 0 00000   TIN
03655  0 00000 0 00000   TO
03656  0 00000 0 00000   TOUT
04263                                  BES 260
04263  0 00000 0 00000   TOP
04264  0 00000 0 00000   TOPO
04265  0 00000 0 00000   TOPP
04266  0 00000 0 00000   TOTAL
04673                                  BES 260
04673  0 00000 0 00000   UNI
05300                                  BES 260
```

```
05300  0 00000 0 00000     UNI2
05301  0 00000 0 00000     X2
05302  0 00000 0 00000     XX
                        *     COMMON BLOCK AS IN HIDECS 2
                 77462              COMMON -1
                 77462     INDIC    COMMON 1
                 77461     ORDER    COMMON 1
                 77460     NWORD    COMMON 1
                 77457     DAT      COMMON 1
                 77456     LGTH     COMMON 1
                 77455     LATIS    COMMON 1
                 77454     NBITH    COMMON 1
                 77453     NBITL    COMMON 1
                 77452     NBIT1    COMMON 1
                 77451     NBIT     COMMON 1
                 77450     NSQ1     COMMON 1
                 77447     OPRMN    COMMON 1
                 77446     ATOMO    COMMON 1
                 77445     ATOM     COMMON 1
                 77444     ONED     COMMON 1
                 77443     D36      COMMON 1
                 77442     ATOOX    COMMON 10
                 77430     ATOX     COMMON 10
                 77416     SET      COMMON 10
                 77404     RANDM    COMMON 10
                 77372     DIFF     COMMON 10
                 77360     CONVT    COMMON 40
                 77310     DATA     COMMON 40
                 77240     MATA     COMMON 40
                 77170     UNIT     COMMON 40
                 77120     COMUN    COMMON 40
                 77050     EQLS     COMMON 20
                 77024     SECTS    COMMON 50
                 76742     MATAX    COMMON 260
                 76336     DROWS    COMMON 2100
                 72252     MROWS    COMMON 2100
                 66166     INMAT    COMMON 5400
                 53536     ATMS     COMMON 2000
                 47616     MACRO    COMMON 7000
                                    END

  LITERALS
05303   000000000000
05304   000000000001
05305   000000000222
05306   777777777777

05307 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM

   NO ERROR IN ABOVE ASSEMBLY
*  DATE AND TIME NOW       32/67 5498.1
```

The decomposition of the sample graph by STABL, follows:

```
 0
14 15 16 17
 0
 0
 0
 0
11 12 13
 0
 6  7  8  9 10
 0
 0
 0
 0
 1  2  3  4  5
 0
 0
 0
```

SIMPX

The best way to visualize SIMPX is as follows.

A graph, since it consists only of vertices (elements) and edges (links), is of course a topological 1-complex. However, we may very easily represent any graph as a many-dimensional complex as follows. A set of vertices has the property that each two vertices in the set are connected by an edge of G, is often called a complete subgraph. Replace each complete subgraph of G by the simplex generated by its vertices. This means that for three points all connected to one another, replace them by a triangle, for four such points, replace with a tetrahedron. Clearly the vertices of the resulting topological complex are precisely the elements of G, and its edges are the links of G.

A most natural decomposition of G, into subsystems, once we have abandoned the idea of trying to decompose it into discrete subsystems, is the decomposition in which each maximal simplex of our topological complex defines a subsystem - namely the set of its vertices.

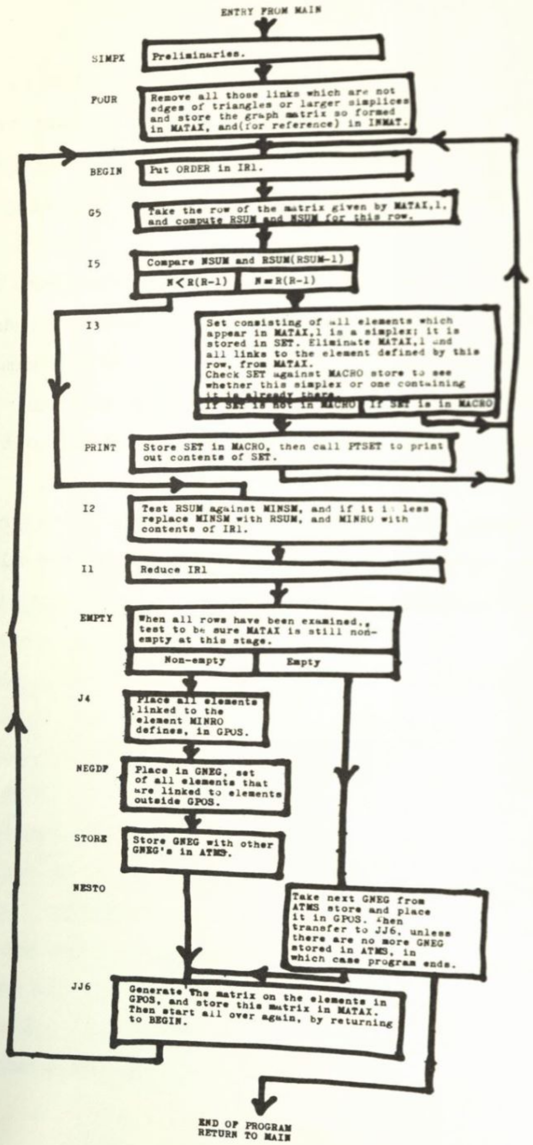
In this case the decomposition presents no intuitive difficulty - in fact, the simplices of G can easily be picked out from the graph by inspection. However, since the task of enumerating all the simplices is rather laborious, it is useful to have it done mechanically. The program SIMPX is based on an algorithm given by Harary and Ross (1957). This algorithm

uses a theorem,which gives a necessary and sufficient condition on the graph matrix, for a vertex to belong to just one simplex. This theorem is only applicable when every simplex has at least 3 vertices. The program therefore begins by removing the isolated links of G.

The algorithm then tests one element at a time, until it finds one which satisfies the condition; when it finds such an element, the program removes it from G, at the same time printing out the unique simplex this vertex defines. Then the program starts again applying the same algorithm to the reduced G.

If no vertex of G is in just one simplex, the graph G is split into two simplex-disjoint subgraphs, and the algorithm applied to these subgraphs in turn.

SIMPX




```
*      FAP
                      00004           ENTRY  SIMPX

  TRANSFER VECTOR
00000   234664456360        COUNT
00001   476362256360        PTSET

  LINKAGE DIRECTOR
00002   000000000000
00003   623144476760

00004 -0634 00 4 00352     SIMPX SXD IR4,4
                           *THIS IS THE UNI GENERATOR
00005  0500 00 0 77461           CLA ORDER
00006  0622 00 0 00024           STD TXLL
00007  0774 00 1 00001           AXT 1,1
00010  0774 00 2 00001           AXT 1,2
00011 -0634 00 2 02404           SXD X2,2
00012  0774 00 4 00001           AXT 1,4
00013 -0500 00 4 77170     CALU  CAL UNIT,4
00014  0602 00 1 01776           SLW UNI,1
00015  0500 00 0 02404           CLA X2
00016  0622 00 1 02403           STD UNI2,1
00017  1 00001 1 00020           TXI *+1,1,1
00020  1 00001 4 00021           TXI *+1,4,1
00021 -3 00044 4 00013           TXL CALU,4,36
00022  1 00001 2 00023           TXI *+1,2,1
00023 -0634 00 2 02404           SXD X2,2
00024 -3 00000 1 00012     TXLL  TXL CALU-1,1,**
                           * NEXT SECTION ERADICATES ALL PAIRS
                           *    FROM MATAX
00025 -0534 00 1 77461           LXD ORDER,1
00026 -0534 00 4 77461           LXD ORDER,4
00027 -0534 00 2 77460     FOUR  LXD NWORD,2
00030  0600 00 0 01371           STZ RINC
00031 -0500 60 1 76742     CIRC  CAL* MATAX,1
00032 -0320 60 4 76742           ANA* MATAX,4
00033 -0765 00 0 00044           LGR 36
00034  0500 00 0 01371           CLA RINC
00035  0522 60 0 00000           XEC* $COUNT
00036  0622 00 0 01371           STD RINC
00037  2 00001 2 00031           TIX CIRC,2,1
00040  0500 00 0 01371           CLA RINC
00041 -0100 00 0 00047           TNZ NEX
00042  0500 00 4 02403           CLA UNI2,4
00043 -0734 00 2 00000           PDX 0,2
00044 -0500 00 4 01776           CAL UNI,4
00045  0760 00 0 00006           COM
00046  0320 60 1 76742           ANS* MATAX,1
00047  2 00001 4 00027     NEX   TIX FOUR,4,1
00050  2 00001 1 00026           TIX FOUR-1,1,1
                           * NEXT SECTION PUTS PERMANENT REFERENCE
                           *  MATRIX,WITHOUT PAIRS) IN INMAT

00051 -0534 00 2 77457           LXD DAT,2
00052 -0500 00 2 72252           CAL MROWS,2
00053  0602 00 2 66166           SLW INMAT,2
```

```
                                     TIX *-2,2,1
00054   2 00001 2 00052       * NEXT SECTION TAKES NEXT NON-ZERO
                              * ROW AND COMPUTES ITS RSUM AND NSUM
00055 -0534 00 1 77461         BEGIN LXD ORDER,1
00056  0500 00 0 02405               CLA =0377777777777
00057  0601 00 0 00353               STO MINSM
00060  0760 00 0 00000         G5    CLM
00061 -0534 00 2 77460               LXD NWORD,2
00062 -0501 60 1 76742               ORA* MATAX,1
00063  2 00001 2 00062               TIX *-1,2,1
00064 -0100 00 0 00066               TNZ *+2
00065  2 00001 1 00060               TIX G5,1,1
00066 -0534 00 4 77461               LXD ORDER,4
00067  0600 00 1 00762               STZ NSUM,1
00070  0600 00 1 01370               STZ RSUM,1
00071  0500 00 4 02403         G4    CLA UNI2,4
00072 -0734 00 2 00000               PDX 0,2
00073 -0500 00 4 01776               CAL UNI,4
00074 -0320 60 1 76742               ANA* MATAX,1
00075  0100 00 0 00112               TZE TIXG4
00076 -0534 00 2 77460               LXD NWORD,2
00077  0600 00 0 01371               STZ RINC
00100 -0500 60 1 76742               CAL* MATAX,1
00101 -0320 60 4 76742               ANA* MATAX,4
00102 -0765 00 0 00044               LGR 36
00103  0500 00 0 01371               CLA RINC
00104  0522 60 0 00000               XEC* $COUNT
00105  0622 00 0 01371               STD RINC
00106  2 00001 2 00100               TIX *-6,2,1
00107  0500 00 0 01371               CLA RINC
00110  0400 00 1 01370               ADD RSUM,1
00111  0601 00 1 01370               STO RSUM,1
00112  2 00001 4 00071         TIXG4 TIX G4,4,1
00113 -0534 00 2 77460               LXD NWORD,2
00114  0560 60 1 76742         G2    LDQ* MATAX,1
00115  0500 00 1 00762               CLA NSUM,1
00116  0522 60 0 00000               XEC* $COUNT
00117  0622 00 1 00762               STD NSUM,1
00120  2 00001 2 00114               TIX G2,2,1
                              * NEXT SECTION TESTS NSUM AND RSUM
                              *    INEQUALITY
00121  0500 00 1 00762         I5    CLA NSUM,1
00122  0100 00 0 00210               TZE I1
00123  0560 00 1 00762               LDQ NSUM,1
00124  0200 00 1 00762               MPY NSUM,1
00125  0767 00 0 00021               ALS 17
00126  0402 00 1 00762               SUB NSUM,1
00127  0340 00 1 01370               CAS RSUM,1
00130  0020 00 0 00202               TRA I2
00131  0020 00 0 00133               TRA I3
A 00132  0000 00 0 00000             HTR
00133 -0534 00 2 77460         I3    LXD NWORD,2
00134 -0500 60 1 76742               CAL* MATAX,1
00135  0602 00 2 77416               SLW SET,2
00136  0600 60 1 76742               STZ* MATAX,1
00137  2 00001 2 00134               TIX *-3,2,1
00140  0500 00 1 02403               CLA UNI2,1
00141 -0734 00 2 00000               PDX 0,2
00142 -0500 00 1 01776               CAL UNI,1
00143 -0602 00 2 77416               ORS SET,2
00144  0760 00 0 00006               COM
00145 -0534 00 4 77461               LXD ORDER,4
00146  0320 60 4 76742               ANS* MATAX,4
00147  2 00001 4 00146               TIX *-1,4,1
                              * NEXT SECTION TESTS TO SEE WHETHER
                              *    ANY SIMPLEX ALREADY  STORED IN MACRO
                              *    CONTAINS SET AS A SUBSET OF ITSELF
                              *    IF SO,  NO PRINT TAKES PLACE
00150 -0534 00 4 00763               LXD OUTIN,4
00151  0600 00 0 00323               STZ CHEK
00152 -0534 00 2 77460         LXN   LXD NWORD,2
00153 -0500 00 4 47616               CAL MACRO,4
00154 -0501 00 2 77416               ORA SET,2
00155  0322 00 4 47616               ERA MACRO,4
00156 -0602 00 0 00323               ORS CHEK
00157  2 00001 4 00160               TIX *+1,4,1
00160  2 00001 2 00153               TIX LXN+1,2,1
00161 -0500 00 0 00323               CAL CHEK
00162  0100 00 0 00055               TZE BEGIN
00163  2 00001 4 00151         PRINT TIX LXN-1,4,1
00164  0500 00 0 00763               CLA OUTIN
00165  0400 00 0 77460               ADD NWORD
00166  0400 00 0 77444               ADD ONED
00167  0622 00 0 00763               STD OUTIN
00170 -0534 00 4 00763               LXD OUTIN,4
00171 -0534 00 2 77460               LXD NWORD,2
00172 -0500 00 2 77416               CAL SET,2
00173  0602 00 4 47616               SLW MACRO,4
00174  2 00001 4 00175               TIX *+1,4,1
00175  2 00001 2 00172               TIX *-3,2,1
00176  0 07400 4 00001               CALL PTSET
00177  1 00000 0 00201
00200  0 10204 0 00002
00201  0020 00 0 00055               TRA BEGIN
00202  0500 00 1 01370         I2    CLA RSUM,1
00203  0340 00 0 00353               CAS MINSM
00204  0020 00 0 00210               TRA *+4
00205  0020 00 0 00210               TRA *+3
00206  0601 00 0 00353               STO MINSM
00207 -0634 00 1 00354               SXD MINRO,1
00210  2 00001 1 00060         I1    TIX G5,1,1
00211 -0534 00 2 77457         EMPTY LXD DAT,2
00212  0760 00 0 00000               CLM
00213 -0501 00 2 72252               ORA MROWS,2
00214  2 00001 2 00213               TIX *-1,2,1
00215  0100 00 0 00261               TZE NESTO
00216  0020 00 0 00217               TRA J4
                              * NEXT SECTION SEPARATES SET OF
                              *    ELEMENTS IN MINRO FROM THE REST
                              *    OF THE MATRIX
00217 -0534 00 1 00354         J4    LXD MINRO,1
00220 -0534 00 2 77460               LXD NWORD,2
```

```
00221 -0500 60 1 76742          CAL* MATAX,1
00222  0602 00 2 00336          SLW GPOS,2
00223  2 00001 2 00221          TIX *-2,2,1
00224  0500 00 1 02403          CLA UNI2,1
00225 -0734 00 2 00000          PDX 0,2
00226 -0500 00 1 01776          CAL UNI,1
00227 -0602 00 2 00336          ORS GPOS,2
                         * NEXT SECTION DEFINES GNEG AS LOGICAL
                         *    OR OF ALL ROWS OF THE MATRIX
                         *    WHOSE ELEMENTS DO NOT APPEAR IN
                         *    GPOS
00230 -0534 00 1 77461   NEGDF  LXD ORDER,1
00231 -0534 00 2 77460          LXD NWORD,2
00232  0600 00 2 00351          STZ GNEG,2
00233  2 00001 2 00232          TIX *-1,2,1
00234  0500 00 1 02403   ONER   CLA UNI2,1
00235 -0734 00 2 00000          PDX 0,2
00236 -0500 00 2 00336          CAL GPOS,2
00237 -0320 00 1 01776          ANA UNI,1
00240 -0100 00 0 00245          TNZ DOWNO
00241 -0534 00 2 77460          LXD NWORD,2
00242 -0500 60 1 76742          CAL* MATAX,1
00243 -0602 00 2 00351          ORS GNEG,2
00244  2 00001 2 00242          TIX *-2,2,1
00245  2 00001 1 00234   DOWNO  TIX ONER,1,1
                         *NEXT SECTION STORES GNEG IN  ATMS
00246  0500 00 0 00355   STORE  CLA NEGIN
00247  0400 00 0 77460          ADD NWORD
00250  0400 00 0 77444          ADD ONED
00251  0622 00 0 00355          STD NEGIN
00252 -0534 00 4 00355          LXD NEGIN,4
00253 -0534 00 2 77460          LXD NWORD,2
00254 -0500 00 2 00351          CAL GNEG,2
00255  0602 00 4 53536          SLW ATMS,4
00256  2 00001 4 00257          TIX *+1,4,1
00257  2 00001 2 00254          TIX *-3,2,1
00260  0020 00 0 00273          TRA JJ6
                         * NEXT SECTION TAKES MOST RECENT
                         *    ADDITION TO ATMS BLOCK AND PUTS
                         *    IT IN GPOS
00261 -0534 00 4 00355   NESTO  LXD NEGIN,4
00262 -3 00001 4 00320          TXL OUT,4,1
00263 -0534 00 2 77460          LXD NWORD,2
00264 -0500 00 4 53536          CAL ATMS,4
00265  0602 00 2 00336          SLW GPOS,2
00266  2 00001 4 00267          TIX *+1,4,1
00267  2 00001 2 00264          TIX *-3,2,1
00270  2 00001 4 00271          TIX *+1,4,1
00271 -0634 00 4 00355          SXD NEGIN,4
00272  0020 00 0 00273          TRA JJ6
                         * NEXT SECTION CREATES NEW MATRIX
                         *    FROM GPOS AND TRANSFERS  TO BEGIN
00273 -0534 00 2 77457   JJ6    LXD DAT,2
00274 -0500 00 2 66166          CAL INMAT,2
00275  0602 00 2 72252          SLW MROWS,2
00276  2 00001 2 00274          TIX *-2,2,1
00277 -0534 00 2 77460          LXD NWORD,2
00300 -0534 00 1 77461          LXD ORDER,1
00301 -0500 00 2 00336          CAL GPOS,2
00302  0320 60 1 76742          ANS* MATAX,1
00303  2 00001 1 00302          TIX *-1,1,1
00304  2 00001 2 00300          TIX *-4,2,1
00305 -0534 00 1 77461          LXD ORDER,1
00306  0500 00 1 02403   CLU    CLA UNI2,1
00307 -0734 00 2 00000          PDX 0,2
00310 -0500 00 2 00336          CAL GPOS,2
00311 -0320 00 1 01776          ANA UNI,1
00312 -0100 00 0 00316          TNZ SKIP
00313 -0534 00 2 77460          LXD NWORD,2
00314  0600 60 1 76742          STZ* MATAX,1
00315  2 00001 2 00314          TIX *-1,2,1
00316  2 00001 1 00306   SKIP   TIX CLU,1,1
00317  0020 00 0 00055          TRA BEGIN
00320 -0534 00 4 00352   OUT    LXD IR4,4
00321  0020 00 4 00001          TRA 1,4
00322  0 00000 0 00000          PZE
00323  0 00000 0 00000   CHEK   PZE
00336                           BES 10
00336  0 00000 0 00000   GPOS
00351                           BES 10
00351  0 00000 0 00000   GNEG
00352  0 00000 0 00000   IR4
00353  0 00000 0 00000   MINSM
00354  0 00000 0 00000   MINRO
00355  0 00000 0 00000   NEGIN
00762                           BES 260
00762  0 00000 0 00000   NSUM
00763  0 00000 0 00000   OUTIN  PZE
01370                           BES 260
01370  0 00000 0 00000   RSUM
01371  0 00000 0 00000   RINC
01776                           BES 260
01776  0 00000 0 00000   UNI
02403                           BES 260
02403  0 00000 0 00000   UNI2
02404  0 00000 0 00000   X2
                         *   COMMON BLOCK FROM HIDECS 2
             77462              COMMON -1
             77462       INDIC  COMMON 1
             77461       ORDER  COMMON 1
             77460       NWORD  COMMON 1
             77457       DAT    COMMON 1
             77456       LGTH   COMMON 1
             77455       LATIS  COMMON 1
             77454       NBITH  COMMON 1
             77453       NBITL  COMMON 1
             77452       NBIT1  COMMON 1
             77451       NBIT   COMMON 1
             77450       NSQ1   COMMON 1
             77447       OPRMN  COMMON 1
             77446       ATOMO  COMMON 1
             77445       ATOM   COMMON 1
```

```
77444   ONED   COMMON 1
77443   D36    COMMON 1
77442   ATODX  COMMON 10
77430   ATOX   COMMON 10
77416   SET    COMMON 10
77404   RANDM  COMMON 10
77372   DIFF   COMMON 10
77360   CONVT  COMMON 40
77310   DATA   COMMON 40
77240   MATA   COMMON 40
77170   UNIT   COMMON 40
77120   COMUN  COMMON 40
77050   EQLS   COMMON 20
77024   SECTS  COMMON 50
76742   MATAX  COMMON 260
76336   DROWS  COMMON 2100
72252   MROWS  COMMON 2100
66166   INMAT  COMMON 5400
53536   ATMS   COMMON 2000
47616   MACRO  COMMON 7000
               END

 LITERALS
02405   377777777777
```

The decomposition of the sample graph follows:

```
14 15 16 17
13 14 15
12 13 15
 7  9 10 12
 7  8  9 10
 6  7  8 10
 4  5  6
 2  4  5
 1  2  5
```


EQCLA

For many purposes the definition of subsystems used in SIMPX is rather restrictive. Thus for instance, suppose we have a set of five elements, connected to one another by 9 links, with just one pair of elements not linked directly. Clearly we should like to consider this set of five elements as a subsystem if it occurs in a larger more diffuse system. But SIMPX will define instead two four element subsystems, and not define the five element system at all.

A second example of a case where SIMPX is too rigid a criterion, is that where the links in the system are very sparse, so that perhaps there are not even any triangles; yet still some elements belong together more tightly than others; we need a weaker definition of subsystem than that used in SIMPX. One way of generalising SIMPX to make it more widely applicable, is given by Luce (1950). In EQCLA, we use another idea.

If we call two vertices x, y of G connected, whenever there exists a sequence of links (xa)(ab)(bc)...(dy), then it is a well known result of graph theory that connectedness is an equivalence relation, and hence that the elements of M fall into disjoint equivalence classes under this relation. These classes are usually called the components of the graph. Now, this is a very weak decomposition criterion; in other words, a completely connected graph has only one component, so the criterion offers us no way of telling anything about what its subsystems might be. It is therefore natural to try to generalise this notion of connectedness.

"Connectedness" is a relation between points, which we might call link-connectedness. Let us introduce a relation between links called triangle-connectedness, a relation between triangles called tetrahedron-connectedness, and so on, where these are defined as follows.

Two links are triangle-connected if there is a sequence of triangles, each overlapping the next in the sequence by two vertices. Similarly two triangles are tetrahedron-connected if there exists a sequence of tetrahedral skeletons, each overlapping the next in the sequence by three vertices. And in general 2 (n-1) simplices are n-connected if there exists a sequence of n-simplices, each overlapping the next in (n-1) vertices.

It is easy to see that just as link-connectedness is an equivalence relation between vertices, so triangle connectedness is an equivalence relation between links, tetrahedron-connectedness is an equivalence relation between triangles, and so on. Each equivalence class of links, under triangle-connectedness defines a set of vertices. We define these sets as the subsystems of triangle-connected elements. Plainly, these subsystems are not necessarily disjoint. Again each

equivalence class of triangles, under tetrahedron-connectedness, defines a set of vertices; we define these sets as the subsystems of tetrahedron-connected elements. These subsystems are again not disjoint; however, each of these subsystems is a subset of some triangle-connected subsystem. (This follows from the fact that two elements cannot be tetrahedron connected, unless they are triangle-connected).

The program EQCLA obtains all the triangle and tetrahedron connected subsystems of G.

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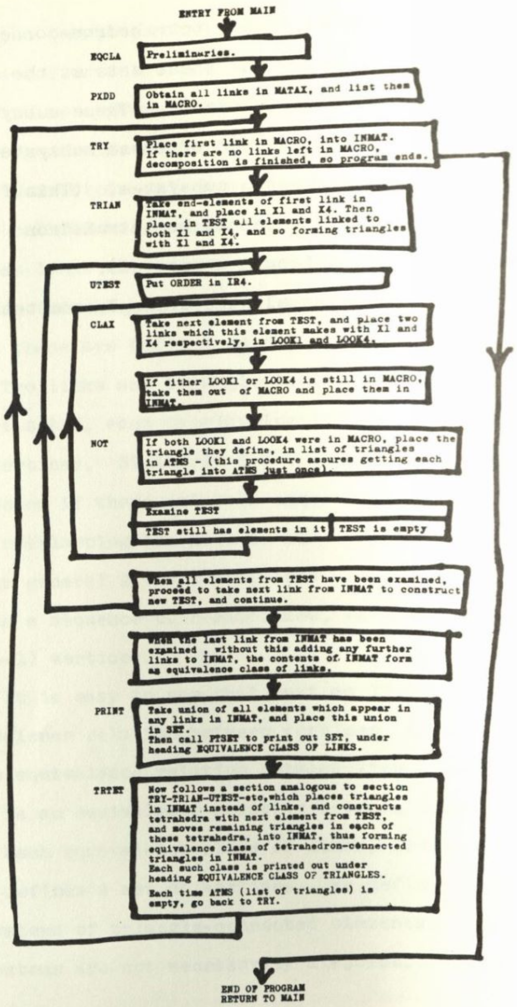
EQCLA




```
*     FAP
                  00005          ENTRY   EQCLA

  TRANSFER VECTOR
00000  635131476360    TRIPT
00001  632563476360    TETPT
00002  476362256360    PTSET

  LINKAGE DIRECTOR
00003  000000000000
00004  255023432160


00005 -0634 00 4 00523   EQCLA SXD IR4,4
                         *THIS IS THE UNI GENERATCR
00006  0500 00 0 77461         CLA ORDER
00007  0622 00 0 00025         STD TXLL
00010  0774 00 1 00001         AXT 1,1
00011  0774 00 2 00001         AXT 1,2
00012 -0634 00 2 01547         SXD X2,2
00013  0774 00 4 00001         AXT 1,4
00014 -0500 00 4 77170   CALU  CAL UNIT,4
00015  0602 00 1 01134         SLW UNI,1
00016  0500 00 0 01547         CLA X2
00017  0622 00 1 01541         STD UNI2,1
00020  1 00001 1 00021         TXI *+1,1,1
00021  1 00001 4 00022         TXI *+1,4,1
00022 -3 00044 4 00014         TXL CALU,4,36
00023  1 00001 2 00024         TXI *+1,2,1
00024 -0634 00 2 01547         SXD X2,2
00025 -3 00000 1 00013   TXLL  TXL CALU-1,1,**
                         * NEXT SECTION LISTS ALL LINKS, ONE
                         *   WORD PER LINK, IN MACRO
00026 -0534 00 1 77461         LXD ORDER,1
00027 -0754 00 1 00000   PXDD  PXD 0,1
00030 -0734 00 4 00000         PDX 0,4
00031  0500 00 4 01541   CLAA  CLA UNI2,4
00032 -0734 00 2 00000         PDX 0,2
00033 -0500 60 1 76742         CAL* MATAX,1
00034 -0320 00 4 01134         ANA UNI,4
00035  0100 00 0 00045         TZE SKIP
00036 -0534 00 2 01542         LXD EDIR,2
00037  0634 00 4 01543         SXA EDGE,4
00040 -0634 00 1 01543         SXD EDGE,1
00041  0500 00 0 01543         CLA EDGE
00042  0601 00 2 47616         STO MACRO,2
00043  1 00001 2 00044         TXI *+1,2,1
00044 -0634 00 2 01542         SXD EDIR,2
00045  2 00001 4 00031   SKIP  TIX CLAA,4,1
00046  2 00001 1 00027         TIX PXDD,1,1
00047  0020 00 0 00050         TRA TRY
                         *END OF SECTION LISTING LINKS IN MACRO
                         *TRY TAKES NEXT NONZERO LINK FROM
                         *   MACRO TO START A NEW EQUIVALENCE
                         *   CLASS IN INMAT
00050 -0534 00 2 01542   TRY   LXD EDIR,2
00051  0600 00 2 66166         STZ INMAT,2
00052  2 00001 2 00051         TIX *-1,2,1
```

```
00053 -0534 00 2 01542          LXD EDIR,2
00054  0500 00 2 47616          CLA MACRO,2
00055 -0100 00 0 00060          TNZ *+3
00056  2 00001 2 00054          TIX *-2,2,1
00057  0020 00 0 00520          TRA OUT
00060  0600 00 2 47616          STZ MACRO,2
00061  0601 00 0 66165          STO INMAT-1
00062  0774 00 2 00002          AXT 2,2
00063 -0634 00 2 01545          SXD SCLIR,2
00064  0774 00 2 00001          AXT 1,2
00065 -0634 00 2 01544          SXD ECLIR,2
00066 -0634 00 2 00525          SXD TRIR,2
00067 -0534 00 2 01544   TRIAN  LXD ECLIR,2
00070  0500 00 2 66166          CLA INMAT,2
00071  1 00001 2 00072          TXI *+1,2,1
00072 -0634 00 2 01544          SXD ECLIR,2
00073  0734 00 4 00000          PAX 0,4
00074 -0634 00 4 01550          SXD X4,4
00075 -0734 00 1 00000          PDX 0,1
00076 -0634 00 1 01546          SXD X1,1
00077 -0534 00 2 77460          LXD NWORD,2
00100 -0500 60 1 76742          CAL* MATAX,1
00101 -0320 60 4 76742          ANA* MATAX,4
00102  0602 00 2 01563          SLW TEST,2
00103  2 00001 2 00100          TIX *-3,2,1
00104  0020 00 0 00105          TRA UTEST
00105 -0534 00 4 77461   UTEST  LXD ORDER,4
00106  0500 00 4 01541   CLAX   CLA UNI2,4
00107 -0734 00 2 00000          PDX 0,2
00110  0500 00 2 01563          CLA TEST,2
00111 -0320 00 4 01134          ANA UNI,4
00112  0100 00 0 00211          TZE NO
00113 -0754 00 4 00000          PXD 0,4
00114  0340 00 0 01546          CAS X1
00115  0020 00 0 00123          TRA LOK1
00116  0000 00 0 00000          HTR
00117  0500 00 0 01546          CLA X1
00120  0622 00 0 01564          STD LOOK1
00121  0634 00 4 01564          SXA LOOK1,4
00122  0020 00 0 00127          TRA LO4
00123 -0634 00 4 01564   LOK1   SXD LOOK1,4
00124  0500 00 0 01546          CLA X1
00125  0771 00 0 00022          ARS 18
00126  0621 00 0 01564          STA LOOK1
00127 -0754 00 4 00000   LO4    PXD 0,4
00130  0340 00 0 01550          CAS X4
00131  0020 00 0 00137          TRA LOK4
00132  0000 00 0 00000          HTR
00133  0500 00 0 01550          CLA X4
00134  0622 00 0 01565          STD LOOK4
00135  0634 00 4 01565          SXA LOOK4,4
00136  0020 00 0 00143          TRA LOKUP
00137 -0634 00 4 01565   LOK4   SXD LOOK4,4
00140  0500 00 0 01550          CLA X4
00141  0771 00 0 00022          ARS 18
00142  0621 00 0 01565          STA LOOK4
00143 -0534 00 2 01542   LOKUP  LXD EDIR,2
00144  0500 00 2 47616          CLA MACRO,2
```

```
00145  0340 00 0 01564          CAS LOOK1
00146  0020 00 0 00150          TRA *+2
00147  0020 00 0 00152          TRA *+3
00150  2 00001 2 00144          TIX *-4,2,1
00151  0020 00 0 00157          TRA REP
00152  0600 00 2 47616          STZ MACRO,2
00153 -0534 00 2 01545          LXD SCLIR,2
00154  0601 00 2 66166          STO INMAT,2
00155  1 00001 2 00156          TXI *+1,2,1
00156 -0634 00 2 01545          SXD SCLIR,2
00157 -0534 00 2 01542   REP    LXD EDIR,2
00160  0500 00 2 47616          CLA MACRO,2
00161  0340 00 0 01565          CAS LOOK4
00162  0020 00 0 00164          TRA *+2
00163  0020 00 0 00166          TRA *+3
00164  2 00001 2 00160          TIX *-4,2,1
00165  0020 00 0 00173          TRA NOT
00166  0600 00 2 47616          STZ MACRO,2
00167 -0534 00 2 01545          LXD SCLIR,2
00170  0601 00 2 66166          STO INMAT,2
00171  1 00001 2 00172          TXI *+1,2,1
00172 -0634 00 2 01545          SXD SCLIR,2
                         * NEXT SECTION PUTS TRIANGLES, IF NEW,
                         *    INTO ATMS ( VERTICES OF TRIANGLES
                         *    APPEAR IN DESCENDING ORDER FROM LEFT
00173 -0754 00 4 00000   NOT    PXD 0,4
00174 -0625 00 0 00453          STL ST
00175  0020 00 0 00410          TRA X14
00176 -0534 00 2 00525          LXD TRIR,2
00177  0500 00 0 00524          CLA TRIWD
00200  0340 00 2 53536   CAST   CAS ATMS,2
00201  0020 00 0 00203          TRA *+2
00202  0020 00 0 00211          TRA NO
00203  0020 00 0 00204          TRA *+1
00204  2 00001 2 00200          TIX CAST,2,1
00205 -0534 00 2 00525          LXD TRIR,2
00206  0601 00 2 53536          STO ATMS,2
00207  1 00001 2 00210          TXI *+1,2,1
00210 -0634 00 2 00525          SXD TRIR,2
                         *END OF SECTION FOR STORING NEW TRIANGLES
00211  2 00001 4 00106   NO     TIX CLAX,4,1
00212  0020 00 0 00213          TRA NEWEC
00213  0500 00 0 01544   NEWEC  CLA ECLIR,
00214  0340 00 0 01545          CAS SCLIR
00215  0020 00 0 00220          TRA *+3
00216  0020 00 0 00221          TRA COMMT
00217  0020 00 0 00220          TRA *+1
00220  0020 00 0 00067          TRA TRIAN
00221  0074 00 4 00000   COMMT  CALL TRIPT
00222  1 00000 0 00224
00223  0 00335 0 00003
00224  0020 00 0 00454          TRA PRINT-3
                         * TRTET TAKES NEXT NONZERO TRIANGLE
                         *   FROM ATMS TO START A NEW EQUIVALENCE
                         *   CLASS OF TRIANGLES
00225 -0534 00 2 00525   TRTET  LXD TRIR,2
00226  0600 00 2 66166          STZ INMAT,2
00227  2 00001 2 00226          TIX *-1,2,1
```

```
00230 -0534 00 2 00525          LXD TRIR,2
00231  0500 00 2 53536          CLA ATMS,2
00232 -0100 00 0 00235          TNZ *+3
00233  2 00001 2 00231          TIX *-2,2,1
                      * TRIANGLES EXHAUSTED, SO GO BACK TO
                      *   NEXT EQUIVALENCE CLASS OF LINKS
00234  0020 00 0 00050          TRA TRY
00235  0600 00 2 53536          STZ ATMS,2
00236  0601 00 0 66165          STO INMAT-1
00237  0774 00 2 00002          AXT 2,2
00240 -0634 00 2 01545          SXD SCLIR,2
00241  0774 00 2 00001          AXT 1,2
00242 -0634 00 2 01544          SXD ECLIR,2
00243 -0534 00 2 01544    TETRA LXD ECLIR,2
00244  0500 00 2 66166          CLA INMAT,2
00245  1 00001 2 00246          TXI *+1,2,1
00246 -0634 00 2 01544          SXD ECLIR,2
00247  0601 00 0 00526          STO HOLD
00250 -0320 00 0 01570          ANA =000077700777
00251  0734 00 4 00000          PAX 0,4
00252 -0734 00 1 00000          PDX 0,1
00253 -0534 00 2 77460          LXD NWORD,2
00254 -0500 60 1 76742          CAL* MATAX,1
00255 -0320 60 4 76742          ANA* MATAX,4
00256  0602 00 2 01563          SLW TEST,2
00257  2 00001 2 00254          TIX *-3,2,1
00260  0500 00 0 00526          CLA HOLD
00261  0771 00 0 00011          ARS 9
00262 -0320 00 0 01570          ANA =000077000777
00263  0734 00 4 00000          PAX 0,4
00264 -0534 00 2 77460          LXD NWORD,2
00265 -0500 00 2 01563          CAL TEST,2
00266 -0320 60 4 76742          ANA* MATAX,4
00267  0602 00 2 01563          SLW TEST,2
00270  2 00001 2 00265          TIX *-3,2,1
00271  0020 00 0 00272          TRA UT
00272 -0534 00 4 77461    UT    LXD ORDER,4
00273  0500 00 4 01541          CLA UNI2,4
00274 -0734 00 2 00000          PDX 0,2
00275  0500 00 2 01563          CLA TEST,2
00276 -0320 00 4 01134          ANA UNI,4
00277  0100 00 0 00375          TZE NOL
00300  0500 00 0 00526          CLA HOLD
00301 -0320 00 0 01571          ANA =000077777000
00302  0622 00 0 01546          STD X1
00303  0767 00 0 00011          ALS 9
00304  0622 00 0 01550          STD X4
00305 -0754 00 4 00000          PXD 0,4
00306 -0625 00 0 00453          STL ST
00307  0020 00 0 00410          TRA X14
00310 -0534 00 2 00525          LXD TRIR,2
00311  0500 00 2 53536          CLA ATMS,2
00312  0340 00 0 00524          CAS TRIWD
00313  0020 00 0 00315          TRA *+2
00314  0020 00 0 00317          TRA *+3
00315  2 00001 2 00311          TIX *-4,2,1
00316  0020 00 0 00324          TRA REP1
00317  0600 00 2 53536          STZ ATMS,2
```

```
                          *     ORDER
00410  0600 00 0 00524    X14   STZ TRIWD
00411  0340 00 0 01546          CAS X1
00412  0020 00 0 00420          TRA AA
00413  0000 00 0 00000          HTR
00414  0340 00 0 01550          CAS X4
00415  0020 00 0 00430          TRA BB
00416  0000 00 0 00000          HTR
00417  0020 00 0 00440          TRA CC
00420 -0602 00 0 00524    AA    ORS TRIWD
00421  0500 00 0 01546          CLA X1
00422  0771 00 0 00011          ARS 9
00423 -0602 00 0 00524          ORS TRIWD
00424  0500 00 0 01550          CLA X4
00425  0771 00 0 00022          ARS 18
00426 -0602 00 0 00524          ORS TRIWD
00427  0020 00 0 00450          TRA STP
00430  0771 00 0 00011    BB    ARS 9
00431 -0602 00 0 00524          ORS TRIWD
00432  0500 00 0 01546          CLA X1
00433 -0602 00 0 00524          ORS TRIWD
00434  0500 00 0 01550          CLA X4
00435  0771 00 0 00022          ARS 18
00436 -0602 00 0 00524          ORS TRIWD
00437  0020 00 0 00450          TRA STP
00440  0771 00 0 00022    CC    ARS 18
00441 -0602 00 0 00524          ORS TRIWD
00442  0500 00 0 01546          CLA X1
00443 -0602 00 0 00524          ORS TRIWD
00444  0500 00 0 01550          CLA X4
00445  0771 00 0 00011          ARS 9
00446 -0602 00 0 00524          ORS TRIWD
00447  0020 00 0 00450          TRA STP
00450  0500 00 0 00453    STP   CLA ST
00451  0400 00 0 01566          ADD =1
00452  0601 00 0 00453          STO ST
00453  0020 00 0 00000    ST    TRA **
                          * THE PRINTING SECTION FOLLOWS,
                          *    PRINTING ENTIRE EQUIVALENCE CLASS
00454 -0534 00 2 77460          LXD NWORD,2
00455  0600 00 2 77416          STZ SET,2
00456  2 00001 2 00455          TIX *-1,2,1
00457 -0534 00 2 01544    PRINT LXD ECLIR,2
00460  0500 00 2 66166          CLA INMAT,2
00461 -0320 00 0 01570          ANA =000077000777
00462  0734 00 4 00000          PAX 0,4
00463  0500 00 4 01541          CLA UNI2,4
00464 -0734 00 2 00000          PDX 0,2
00465 -0500 00 4 01134          CAL UNI,4
00466 -0602 00 2 77416          ORS SET,2
00467 -0534 00 2 01544          LXD ECLIR,2
00470  0500 00 2 66166          CLA INMAT,2
00471  0771 00 0 00011          ARS 9
00472 -0320 00 0 01570          ANA =000077000777
00473  0734 00 4 00000          PAX 0,4
00474  0500 00 4 01541          CLA UNI2,4
00475 -0734 00 2 00000          PDX 0,2
00476 -0500 00 4 01134          CAL UNI,4
```

```
00320 -0534 00 2 01545        LXD SCLIR,2
00321  0601 00 2 66166        STO INMAT,2
00322  1 00001 2 00323        TXI *+1,2,1
00323 -0634 00 2 01545        SXD SCLIR,2
00324  0500 00 0 00526  REP1  CLA HOLD
00325 -0320 00 0 01570        ANA =O000777000777
00326  0622 00 0 01546        STD X1
00327  0767 00 0 00022        ALS 18
00330  0622 00 0 01550        STD X4
00331 -0754 00 4 00000        PXD 0,4
00332 -0625 00 0 00453        STL ST
00333  0020 00 0 00410        TRA X14
00334 -0534 00 2 00525        LXD TRIR,2
00335  0500 00 2 53536        CLA ATMS,2
00336  0340 00 0 00524        CAS TRIWD
00337  0020 00 0 00341        TRA *+2
00340  0020 00 0 00343        TRA *+3
00341  2 00001 2 00335        TIX *-4,2,1
00342  0020 00 0 00350        TRA REP2
00343  0600 00 2 53536        STZ ATMS,2
00344 -0534 00 2 01545        LXD SCLIR,2
00345  0601 00 2 66166        STO INMAT,2
00346  1 00001 2 00347        TXI *+1,2,1
00347 -0634 00 2 01545        SXD SCLIR,2
00350  0500 00 0 00526  REP2  CLA HOLD
00351 -0320 00 0 01567        ANA =O000000777777
00352  0767 00 0 00011        ALS 9
00353  0622 00 0 01546        STD X1
00354  0767 00 0 00011        ALS 9
00355  0622 00 0 01550        STD X4
00356 -0754 00 4 00000        PXD 0,4
00357 -0625 00 0 00453        STL ST
00360  0020 00 0 00410        TRA X14
00361 -0534 00 2 00525        LXD TRIR,2
00362  0500 00 2 53536        CLA ATMS,2
00363  0340 00 0 00524        CAS TRIWD
00364  0020 00 0 00366        TRA *+2
00365  0020 00 0 00370        TRA *+3
00366  2 00001 2 00362        TIX *-4,2,1
00367  0020 00 0 00375        TRA NOL
00370  0600 00 2 53536        STZ ATMS,2
00371 -0534 00 2 01545        LXD SCLIR,2
00372  0601 00 2 66166        STO INMAT,2
00373  1 00001 2 00374        TXI *+1,2,1
00374 -0634 00 2 01545        SXD SCLIR,2
00375  2 00001 4 00273  NOL   TIX UT+1,4,1
00376  0500 00 0 01544        CLA ECLIR
00377  0340 00 0 01545        CAS SCLIR
00400  0020 00 0 00403        TRA *+3
00401  0020 00 0 00404        TRA COMT
00402  0020 00 0 00403        TRA *+1
00403  0020 00 0 00243        TRA TETRA
00404  0074 00 4 00001  COMT  CALL TETPT
00405  1 00000 0 00407
00406  0 00624 0 00003
00407  0020 00 0 00454        TRA PRINT-3
                        * SUBROUTINE FOR GETTING VERTICES WITHIN
                        *   A WORD IN DESCENDING NUMERICAL
```

```
00477 -0602 00 2 77416        ORS SET,2
00500 -0534 00 2 01544        LXD ECLIR,2
00501  0500 00 2 66166        CLA INMAT,2
00502 -0734 00 4 00000        PDX 0,4
00503  0500 00 4 01541        CLA UNI2,4
00504 -0734 00 2 00000        PDX 0,2
00505 -0500 00 4 01134        CAL UNI,4
00506 -0602 00 2 77416        ORS SET,2
00507  0500 00 0 01544        CLA ECLIR
00510  0402 00 0 77444        SUB ONED
00511  0100 00 0 00514        TZE CALLP
00512  0601 00 0 01544        STO ECLIR
00513  0020 00 0 00457        TRA PRINT
00514  0074 00 4 00002  CALLP CALL PTSET
00515  1 00000 0 00517
00516  0 01002 0 00003
00517  0020 00 0 00225        TRA TRTET
00520 -0534 00 4 00523  OUT   LXD IR4,4
00521  0020 00 4 00001        TRA 1,4
00522  0 00000 0 00000        PZE
00523  0 00000 0 00000  IR4
00524  0 00000 0 00000  TRIWD
00525  0 00000 0 00000  TRIR
00526  0 00000 0 00000  HOLD
00527  0 00000 0 00000  NEWT
01134                         BES 260
01134  0 00000 0 00000  UNI
01541                         BES 260
01541  0 00000 0 00000  UNI2
01542  0 00000 0 00000  EDIR
01543  0 00000 0 00000  EDGE
01544  0 00000 0 00000  ECLIR
01545  0 00000 0 00000  SCLIR
01546  0 00000 0 00000  X1
01547  0 00000 0 00000  X2
01550  0 00000 0 00000  X4
01563                         BES 10
01563  0 00000 0 00000  TEST
01564  0 00000 0 00000  LOOK1
01565  0 00000 0 00000  LOOK4
                        *     COMMON BLOCK FROM HIDECS 2
                 77462        COMMON -1
                 77462  INDIC COMMON 1
                 77461  ORDER COMMON 1
                 77460  NWORD COMMON 1
                 77457  DAT   COMMON 1
                 77456  LGTH  COMMON 1
                 77455  LATIS COMMON 1
                 77454  NBITH COMMON 1
                 77453  NBITL COMMON 1
                 77452  NBIT1 COMMON 1
                 77451  NBIT  COMMON 1
                 77450  NSQ1  COMMON 1
                 77447  OPRMN COMMON 1
                 77446  ATOMO COMMON 1
                 77445  ATOM  COMMON 1
                 77444  ONED  COMMON 1
                 77443  D36   COMMON 1
```

77442	ATOOX	COMMON	10
77430	ATCX	COMMON	10
77416	SET	COMMON	10
77404	RANDM	COMMON	10
77372	DIFF	COMMON	10
77360	CONVT	COMMON	40
77310	DATA	COMMON	40
77240	MATA	COMMON	40
77170	UNIT	COMMON	40
77120	COMUN	COMMON	40
77050	EQLS	COMMON	20
77024	SECTS	COMMON	50
76742	MATAX	COMMON	260
76336	DROWS	COMMON	2100
72252	MROWS	COMMON	2100
66166	INMAT	COMMON	5400
53536	ATMS	COMMON	2000
47616	MACRO	COMMON	7000
		END	

LITERALS

01566	000000000001
01567	000000777777
01570	000777000777
01571	000777777000

01572 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM

The decomposition of the sample graph follows:

EQUIVALENCE CLASS OF LINKS
1 2 3 4 5 6
EQUIVALENCE CLASS OF TRIANGLES
4 5 6
EQUIVALENCE CLASS OF TRIANGLES
2 3 4
EQUIVALENCE CLASS OF TRIANGLES
2 4 5
EQUIVALENCE CLASS OF TRIANGLES
1 2 3
EQUIVALENCE CLASS OF TRIANGLES
1 2 5
EQUIVALENCE CLASS OF LINKS
6 7 8 9 10 11
EQUIVALENCE CLASS OF TRIANGLES
6 7 8 9 10 11
EQUIVALENCE CLASS OF LINKS
4 11
EQUIVALENCE CLASS OF LINKS
11 12
EQUIVALENCE CLASS OF LINKS
12 13 14 15 16 17
EQUIVALENCE CLASS OF TRIANGLES
14 15 16 17
EQUIVALENCE CLASS OF TRIANGLES
13 14 15
EQUIVALENCE CLASS OF TRIANGLES
12 13 15
EQUIVALENCE CLASS OF LINKS
4 16
EQUIVALENCE CLASS OF LINKS
1 17

CONCLUSION

All the four subprograms described are under the control of the program MAIN, which follows. There also follow listings of the two subsidiary subprograms, TETPT, and PTSET, which these programs require. All the other programs called by MAIN, BIDUP, STABL, SIMPX, EQCLA, are described, in detail, in the HIDECS 2 report. They are: INPAR, GENER, INDAT, CNDAT, SYMET, PTMAT, COUNT-CNVRT, PTLVL. The machine specification is the same as for HIDECS 2; IBM 7090, 32K core storage, MIT-FMS system; each of the four programs can handle systems of up to 250 elements. Finally, note that it is not possible to run more than one of these programs on the same calling sequence, because there is only room in core for storing one at a time.

```
77442  ATOOX  COMMON 10
77430  ATCX   COMMON 10
77416  SET    COMMON 10
77404  RANDM  COMMON 10
77372  DIFF   COMMON 10
77360  CONVT  COMMON 40
77310  DATA   COMMON 40
77240  MATA   COMMON 40
77170  UNIT   COMMON 40
77120  COMUN  COMMON 40
77050  EQLS   COMMON 20
77024  SECTS  COMMON 50
76742  MATAX  COMMON 260
76336  DROWS  COMMON 2100
72252  MROWS  COMMON 2100
66166  INMAT  COMMON 5400
53536  ATMS   COMMON 2000
47616  MACRO  COMMON 7000
              END

  LITERALS
01566  000000000001
01567  000000777777
01570  000777000777
01571  000777777000

01572 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM
```

The decomposition of the sample graph follows:

```
EQUIVALENCE CLASS OF LINKS
   1  2  3  4  5  6
EQUIVALENCE CLASS OF TRIANGLES
   4  5  6
EQUIVALENCE CLASS OF TRIANGLES
   2  3  4
EQUIVALENCE CLASS OF TRIANGLES
   2  4  5
EQUIVALENCE CLASS OF TRIANGLES
   1  2  3
EQUIVALENCE CLASS OF TRIANGLES
   1  2  5
EQUIVALENCE CLASS OF LINKS
   6  7  8  9 10 11
EQUIVALENCE CLASS OF TRIANGLES
   6  7  8  9 10 11
EQUIVALENCE CLASS OF LINKS
   4 11
EQUIVALENCE CLASS OF LINKS
  11 12
EQUIVALENCE CLASS OF LINKS
  12 13 14 15 16 17
EQUIVALENCE CLASS OF TRIANGLES
  14 15 16 17
EQUIVALENCE CLASS OF TRIANGLES
  13 14 15
EQUIVALENCE CLASS OF TRIANGLES
  12 13 15
EQUIVALENCE CLASS OF LINKS
   4 16
EQUIVALENCE CLASS OF LINKS
   1 17
```


CONCLUSION

All the four subprograms described are under the control of the program MAIN, which follows. There also follow listings of the two subsidiary subprograms, TETPT, and PTSET, which these programs require. All the other programs called by MAIN, BLDUP, STABL, SIMPX, EQCLA, are described, in detail, in the HIDECS 2 report. They are: INPAR, GENER, INDAT, CNDAT, SYMET, PTMAT, COUNT-CNVRT, PTLVL. The machine specification is the same as for HIDECS 2; IBM 7090, 32K core storage, MIT-FMS system; each of the four programs can handle systems of up to 250 elements. Finally, note that it is not possible to run more than one of these programs on the same calling sequence, because there is only room in core for storing one at a time.

```
 THIS IS MAIN PROGRAM
*     FAP
 TRANSFER VECTOR
00000  336225636447      .SETUP
00001  626346442147      STOMAP
00002  314547215160      INPAR
00003  272545255160      GENER
00004  314524216360      INDAT
00005  234524216360      CNDAT
00006  627044256360      SYMET
00007  476344216360      PTMAT
00010  626321224360      STABL
00011  256731636060      EXIT

00012  0074 00 4 00000            TSX $.SETUP,4
00013  0074 00 4 00001            CALL STOMAP
00014  1 00000 0 00016
00015  0 00015 0 00000
00016  0074 00 4 00002            CALL INPAR
00017  1 00000 0 00021
00020  0 00020 0 00000
00021  0074 00 4 00003            CALL GENER
00022  1 00000 0 00024
00023  0 00025 0 00000
00024  0074 00 4 00004            CALL INDAT
00025  1 00000 0 00027
00026  0 00030 0 00000
00027  0074 00 4 00005            CALL CNDAT
00030  1 00000 0 00032
00031  0 00033 0 00000
00032  0074 00 4 00006            CALL SYMET
00033  1 00000 0 00035
00034  0 00040 0 00000
00035  0074 00 4 00007            CALL PTMAT
00036  1 00000 0 00040
00037  0 00043 0 00000
00040  0074 00 4 00010            CALL STABL
00041  1 00000 0 00043
00042  0 00050 0 00000
                         * THE PREVIOUS CARD MAY BE REPLACED
                         *   BY CALL BLDUP, CALL SIMPX, OR
                         *   CALL EQCLA, WHICHEVER IS REQUIRED
                         * IT IS NOT POSSIBLE TO CALL MORE THAN
                         *   ONE OF THEM IN THE SAME CALLING
                         *   SEQUENCE, SINCE THERE IS NOT
                         *   ENOUGH ROOM IN CORE TO STORE THE
                         *   PROGRAMS THEMSELVES MORE THAN
                         *   ONE AT A TIME
00043  0074 00 4 00011            CALL EXIT
00044  1 00000 0 00046
00045  0 00053 0 00000

                         *   COMMON BLOCK FROM HIDECS 2
                 77462          COMMON -1
                 77462    INDIC COMMON 1
                 77461    ORDER COMMON 1
                 77460    NWORD COMMON 1
                 77457    DAT   COMMON 1
```

```
THIS IS MAIN PROGRAM

                 77456    LGTH   COMMON 1
                 77455    LATIS  COMMON 1
                 77454    NBITH  COMMON 1
                 77453    NBITL  COMMON 1
                 77452    NBIT1  COMMON 1
                 77451    NBIT   COMMON 1
                 77450    NSQ1   COMMON 1
                 77447    OPRMN  COMMON 1
                 77446    ATOMO  COMMON 1
                 77445    ATOM   COMMON 1
                 77444    ONED   COMMON 1
                 77443    D36    COMMON 1
                 77442    ATOOX  COMMON 10
                 77430    ATOX   COMMON 10
                 77416    SET    COMMON 10
                 77404    RANDM  COMMON 10
                 77372    DIFF   COMMON 10
                 77360    CONVT  COMMON 40
                 77310    DATA   COMMON 40
                 77240    MATA   COMMON 40
                 77170    UNIT   COMMON 40
                 77120    COMUN  COMMON 40
                 77050    EQLS   COMMON 20
                 77024    SECTS  COMMON 50
                 76742    MATAX  COMMON 260
                 76336    DROWS  COMMON 2100
                 72252    MROWS  COMMON 2100
                 66166    INMAT  COMMON 5400
                 53536    ATMS   COMMON 2000
                 47616    MACRO  COMMON 7000
                                 END

00046 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM

   NO ERROR IN ABOVE ASSEMBLY
* DATE AND TIME NOW      32/67 5498.1
```

```
*    FAP

                    00011          ENTRY   TETPT
                    00006          ENTRY   TRIPT

                         *  COMMENTS IN THE PROGRAM EQCLA

  TRANSFER VECTOR
00000  746263303460     (STH)
00001  742631433460     (FIL)

  LINKAGE DIRECTOR
00002  000000000000
00003  632563476360

00004   0 00000 0 00000          PZE
00005   0 00000 0 00013          PZE THREE
00006   0500 00 0 00005    TRIPT CLA *-1
00007   0020 00 0 00027          TRA START
00010   0 00000 0 00021          PZE FOUR
00011   0500 00 0 00010    TETPT CLA *-1
00012   0020 00 0 00027          TRA START
00013  602550643165        THREE BCI 6, EQUIVALENCE CLASS OF LINKS
00014  214325452325
00015  602343216262
00016  604626604331
00017  454262606060
00020  606060606060
00021  602550643165        FOUR  BCI 6, EQUIVALENCE CLASS OF TRIANGLES
00022  214325452325
00023  602343216262
00024  604626606351
00025  312145274325
00026  626060606060
00027   0621 00 0 00037   START  STA LST1
00030  -0634 00 4 00051          SXD IR4,4
00031  -0500 00 0 00046          CAL TAPE2
00032   0074 00 4 00000          TSX $(STH),4
00033   0 00000 0 00047          PZE LELFT
00034  -0500 00 0 00050          CAL NUM
00035   0622 00 0 00042          STD TXH1
00036   0774 00 1 00000          AXT 0,1
00037   0560 00 1 00000   LST1   LDQ **,1
00040  -1 00000 0 00000          STR
00041   1 77777 1 00042          TXI *+1,1,-1
00042   3 00000 1 00037   TXH1   TXH LST1,1,**
00043   0074 00 4 00001          TSX $(FIL),4
00044  -0534 00 4 00051          LXD IR4,4
00045   0020 00 4 00001          TRA 1,4
00046   0 00002 0 00000   TAPE2  PZE 0,0,2
00047  740621063460       LELFT  BCI 1,(6A6)
00050   0 77772 0 00000   NUM    PZE 0,0,-6
00051   0 00000 0 00000   IR4
                        *   COMMON BLOCK FROM HIDECS 2
               77462             COMMON -1
               77462      INDIC  COMMON 1
               77461      ORDER  COMMON 1
               77460      NWORD  COMMON 1
               77457      DAT    COMMON 1
               77456      LGTH   COMMON 1
               77455      LATIS  COMMON 1
               77454      NBITH  COMMON 1
               77453      NBITL  COMMON 1
               77452      NBIT1  COMMON 1
               77451      NBIT   COMMON 1
               77450      NSQ1   COMMON 1
               77447      OPRMN  COMMON 1
               77446      ATOMO  COMMON 1
               77445      ATOM   COMMON 1
               77444      ONED   COMMON 1
               77443      D36    COMMON 1
               77442      ATOOX  COMMON 10
               77430      ATOX   COMMON 10
               77416      SET    COMMON 10
               77404      RANDM  COMMON 10
               77372      DIFF   COMMON 10
               77360      CONVT  COMMON 40
               77310      DATA   COMMON 40
               77240      MATA   COMMON 40
               77170      UNIT   COMMON 40
               77120      COMUN  COMMON 40
               77050      EQLS   COMMON 20
               77024      SECTS  COMMON 50
               76742      MATAX  COMMON 260
               76336      DROWS  COMMON 2100
               72252      MROWS  COMMON 2100
               66166      INMAT  COMMON 5400
               53536      ATMS   COMMON 2000
               47616      MACRO  COMMON 7000
                                 END
```

```
*       FAP

                      CC004          ENTRY PTSET

   TRANSFER VECTOR
 00000   746263303460        (STH)
 00001   742631433460        (FIL)

   LINKAGE DIRECTCR
 00002   00000000CCCC
 00003   476362256360

 00004 -0634 00 4 CC710   PTSET SXD IR4,4
                        *            PRELIMINARY
 00005   050C C0 0 77461          CLA ORDER
 00006   0622 00 0 CC017          STD TXL1
 00007   0622 CC 0 CCC43          STD NOK
 C0010   0500 00 0 77460          CLA NWORD
 00011   0622 00 0 C0034          STD TXL3
                        *            GENERATE TABLE OF NUMBERS
 00012   C774 OC 1 CCC01          AXT 1,1
 00013   0500 C0 0 77444          CLA ONED
 00014   0622 C0 1 CC707          STD TABLE,1
 00015   0400 00 0 77444          ADD ONED
 00016   1 00001 1 CC017          TXI *+1,1,1
 C0017 -3 00000 1 CCC14   TXL1    TXL *-3,1,**
                        *            MODIFY TABLE TO INDICATE SET
 00020   0774 00 2 CC001          AXT 1,2
 00021   0774 00 4 CC001          AXT 1,4
 00022   0774 00 1 0C001   AXT1   AXT 1,1
 00023   0560 00 2 77416          LDC SET,2
 00024   0162 00 0 00026          TQP *+2
 00025   0020 00 0 CCC27          TRA *+2
 00026   060C 00 4 C0707   STORE  STZ TABLE,4
 00027 -0773 00 0 CCCC1           RQL 1
 00030   1 00001 1 CC031          TXI *+1,1,1
 00031   1 00001 4 CCC32          TXI *+1,4,1
 00032 -3 00044 1 CC024           TXL STORE-2,1,36
 00033   1 00001 2 0C034          TXI *+1,2,1
 00034 -3 00C00 2 0C022    TXL3   TXL AXT1,2,**
 00035   0774 00 4 CCC01          AXT 1,4
 00036   0774 00 1 0C001          AXT 1,1
 00037   0500 00 1 00707   CLAT   CLA TABLE,1
 C0040   0100 00 0 C0C43          TZE NOK
 00041   0622 00 4 0C707          STD TABLE,4
 00042   1 C0C01 4 00C43          TXI *+1,4,1
 00043   3 00000 1 00045   NOK    TXH FORM,1,**
 00044   1 00001 1 CC037          TXI CLAT,1,1
 00045   2 00001 4 CC046   FORM   TIX *+1,4,1
 00046 -0634 00 4 0CC57           SXD TXL2,4
 00047   0020 00 0 CC050          TRA OUT
                        *            PRINT OUT MODIFIED TABLE.
 00050 -0500 00 0 CC064    OUT    CAL NN
 00051   0074 00 4 CCC0C          TSX $(STH),4
 00052   0 00000 0 0C066          PZE FMT
 00053   0774 00 1 0CC01          AXT 1,1
 00054   0560 00 1 CC707   LST2   LDQ TABLE,1
 00055 -1 00000 0 0CC00           STR
 00056   1 00001 1 0C057          TXI *+1,1,1
 00057 -3 00000 1 0C054    TXL2   TXL LST2,1,**
 00060   0074 00 4 00001          TSX $(FIL),4
 00061   0020 00 0 CC062          TRA FINIS
 00062 -0534 00 4 CC710    FINIS  LXD IR4,4
 00063   0020 00 4 CCC01          TRA 1,4
 00064   0 00002 0 0C000   NN     PZE 0,0,2
 00065   C6310334606C             BCI 1,6I3)
 00066   740130607303      FMT    BCI 1,(1H ,3
 00707                            BES 400
 00707   0 C0000 0 0CC00   TABLE
 00710   0 00000 0 0CC00   IR4
 00711   0 00000 C 0CC00   TWO
                        *   COMMON BLOCK FROM HIDECS 2
                 77462            COMMON -1
                 77462     INDIC  COMMON 1
                 77461     ORDER  COMMON 1
                 77460     NWORD  COMMON 1
                 77457     DAT    COMMON 1
                 77456     LGTH   COMMON 1
                 77455     LATIS  COMMON 1
                 77454     NBITH  COMMON 1
                 77453     NBITL  COMMON 1
                 77452     NBIT1  COMMON 1
                 77451     NBIT   COMMON 1
                 77450     NSQ1   COMMON 1
                 77447     OPRNN  COMMON 1
                 77446     ATOMO  COMMON 1
                 77445     ATOM   COMMON 1
                 77444     ONED   COMMON 1
                 77443     D36    COMMON 1
                 77442     ATOOX  COMMON 10
                 77430     ATOX   COMMON 10
                 77416     SET    COMMON 10
                 77404     RANDM  COMMON 10
                 77372     DIFF   COMMON 10
                 77360     CONVT  COMMON 40
                 77310     DATA   COMMON 40
                 77240     MATA   COMMON 40
                 77170     UNIT   COMMON 40
                 77120     COMUN  COMMON 40
                 77050     EQLS   COMMON '20
                 77024     SECTS  COMMON 50
                 76742     MATAX  COMMON 260
                 76336     DROWS  COMMON 2100
                 72252     MROWS  COMMON 2100
                 66166     INMAT  COMMON 5400
                 53536     ATMS   COMMON 2000
                 47616     MACRO  COMMON 7000
                                  END
```

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HIDECS 3:
FOUR COMPUTER PROGRAMS FOR THE HIERARCHICAL DECOMPOSITION OF SYSTEMS WHICH HAVE AN ASSOCIATED LINEAR GRAPH

Christopher Alexander,

63-27

June, 1963

MIT

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