

ON CHANGING THE WAY PEOPLE SEE¹

CHRISTOPHER ALEXANDER²

AND

A. W. F. HUGGINS³

*Society of Fellows
Harvard University*

*Center for Cognitive Studies
Harvard University*

Summary.—Ss see black and white linear patterns either as sequences ("sequentially"), or as wholes ("figurally"). The object of this study was to determine under which conditions, if any, Ss can be brought to change their perceptions from "sequential" to "figural." (1) Ss tended to see the patterns sequentially at first, but (2) the task of searching for single patterns in disorderly arrays of many patterns forced Ss to see the single patterns figurally. (3) Other kinds of experience, expected to have the same effect, induced little change.

It is well known that certain linear patterns are first perceived as sequences, and only later, when the perceiver has more experience, seen as "wholes." Examples occur in music, language, morse-code, telegraphy, painting, arithmetic, the analysis of sequential data from an experiment. In all cases a naive perceiver hears, reads, or sees step by step, building up the pattern from very small units, and is at first unable to appreciate or even sense the larger whole. As the perceiver becomes experienced, he is able to take in larger units at a time, and thereby get a better sense of the whole organization—so much better, indeed, that the ability to see large patterns in such a sequence is always taken as a mark of sophistication.

It is naturally important to know how to induce this sophistication artificially. In particular, it is important to invent ways of inducing it that do not rely on specific demonstrations of what the larger units are, but somehow force the perceiver to become aware of them by himself.

The experiments presented here deal with the visual perception of 35 black and white linear patterns, and three ways of forcing Ss to perceive these patterns as wholes.

To decide whether experience changes S's mode of perception, we must first find some objective means of describing the way in which S sees the patterns. The obvious ways of doing this are based on similarity judgments, on confusions, or on concept formation experiments (Alexander, 1960; Heider, 1924; Hull, 1920; Miller & Nicely, 1955; Osgood, Suci, & Tannenbaum, 1957). However, all these methods have two serious drawbacks. (1) They require a large number of judgments, in fact so many that S's mode of perception may change during the course of the experiment. (2) The results still

¹This investigation was supported in part by a grant from the National Science Foundation No. NSF Grant G-16486 (period 1/30/61-1/29/64) to Harvard University, Center for Cognitive Studies.

²Now at Department of Architecture, University of California, Berkeley.

³Now at Research Laboratory of Electronics, M.I.T.

need interpretation—through factor analysis, clustering techniques, or guessing.

We believe that we have discovered a much easier and neater way of describing *S*'s manner of seeing. If *S* lays all 35 of our stimulus patterns on a board, and arranges them to make them as easy as possible for him to find, the board soon has a definite, orderly character. But different *S*s by no means agree about the kind of order or arrangement which works best. Indeed, we find that each *S* has his own way of doing it.

The arrangement *S* makes on his board gives us a beautifully explicit account of the way he perceives the patterns. He groups together those patterns which he perceives as similar.

EXPERIMENTAL ARRANGEMENTS

Materials

For stimuli we used horizontal rectangular patterns, each one in effect a strip composed of three black and four white squares. Adjacent squares of the same color were not separated. There were 35 different patterns of this kind [$7!/(3! \times 4!)$]. They are illustrated in Fig. 1. They were made from high-contrast photostatic copies of black and white card masters. Each pattern was $\frac{3}{8}$ in. high, and $2\frac{3}{8}$ in. long; it was protected by a strip of transparent "Scotch" mending tape which overlapped the pattern on the bottom edge so that the bottom of each pattern was marked, clearly but unobtrusively, by a greyish semi-transparent strip.

These patterns were always seen against a board whose top surface was an achromatic grey paper. The paper was selected from those available so that neither the black nor the white portions of the patterns seemed to stand out more clearly than the other. (The reflectance of the white was about 0.9, that of the grey about 0.3, that of the black about 0.05). The board was 18 in. by 24 in., with a grid drawn on it in pencil. This grid contained 7 columns and 15 rows, making 105 rectangular cells, each 1 in. high by $3\frac{1}{4}$ in. wide. The cell size was chosen so that patterns placed in adjacent cells were close enough to be taken in at one glance, but not so close that they interfered with one another visually.

The third item of equipment was a set of 35 different slides. Each slide was a photograph of one pattern, lying on the achromatic grey paper mentioned above. The slides were back-projected on a milk-glass screen so that a pattern, projected from the slide onto this screen, looked like a pattern seen on the board, both in size and over-all brightness.

The illumination of the board remained constant throughout the experiments, to avoid any difference in black-white-grey relationships affecting the results.⁴ The 35 slides were in the projector in a fixed order, originally determined by a random process, but left unchanged throughout the experiments.

⁴The photographs are misleading in this respect. They were taken under conditions of uneven illumination, different from the laboratory conditions.

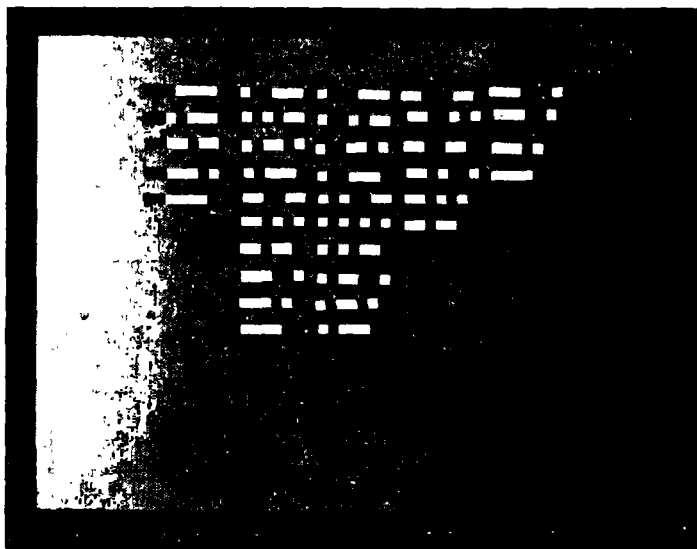


FIG. 1a. Sequential example

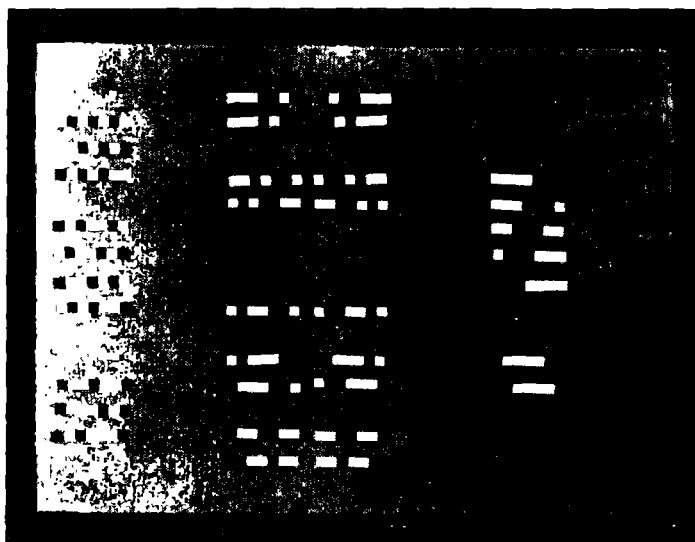


FIG. 1b. Figural example

The fourth item to be described is what we call a random-like array. This is an arrangement of the 35 patterns in a rectangular block, 5 cells wide, by 7 cells high, in which there is no discernible order or regularity. An arrangement generated by a random process would of course contain all kinds of minor regularities. The random-like array was made by laying the patterns out in a rec-

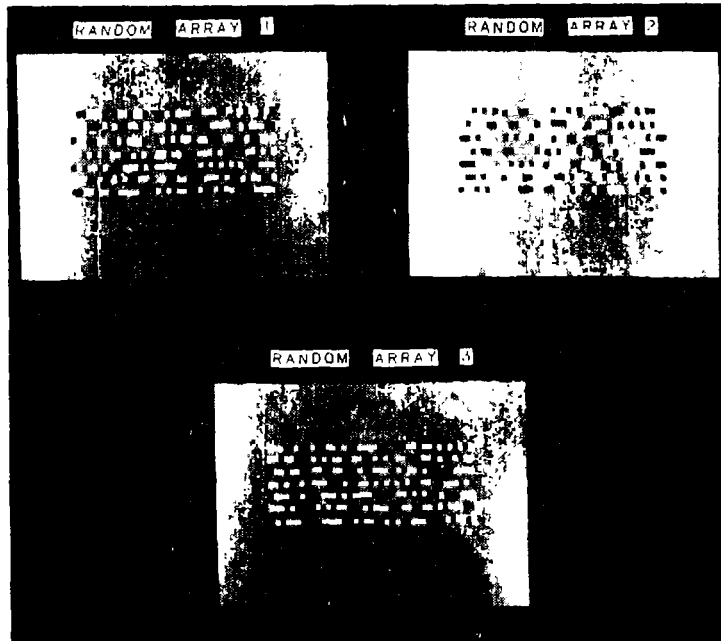


FIG. 2. Random arrays

tangular block five columns wide. Each relationship that could be seen was then destroyed, until there were no orderly connections between any pair of patterns in adjacent cells. The random-like arrays are shown in Fig. 2.

Subjects

Ss for all the experiments to be described were undergraduate girls from Radcliffe College. None of them had seen this material before.

Test Procedure

When *S* came in she sat down at a table. The ground glass screen was in front of her, about 2½ ft. away, and beyond it was the projector. Between her and the screen, on the table, was the board. On it a random array (No. 1) was laid out. As soon as she sat down she was given the following instructions.

Instructions for test.—There are 35 patterns here, all different. I am going to show one of these patterns to you at a time, there on the screen, and as soon as I show it to you I want you to find the pattern among those on the board. All the patterns I shall show you will be on the board; there are no tricks. I may show you the same pattern more than once, or even several times.

Every time I put a new one on the screen, I shall start this clock, and I want you to find the corresponding one on your board as fast as you can. It must be the same way up as the one on the screen. When you find it point to it. When you point to the right one, I shall stop the clock.

Now as you see, the patterns are loose and may be moved (demonstrating). Each time after I stop the clock, you will have about 15 or 20 sec. of free time. During this time you may move the patterns if you wish so as to make it as easy as possible for you to find the ones I show you. You may put them wherever you wish, except for two things. (1) You must keep all the patterns the same way up that they are now. (2) You must always place them one per compartment in the middle of the compartments marked on the board.

You need not keep them in the tight rectangular array they are in now. In fact, if you do you'll probably make things rather hard for yourself.

As soon as *S* understood the instructions, the test began, so as to give her no time to examine the random array and develop any special kind of set due to preconceptions. *E* projected the first pattern on the screen by a remote control connected to the automatic slide projector which carried the tray of all 35 slides. When the slide came onto the screen, *E* started a clock. The pattern remained on the screen until *S* found it. If *S* pointed to the wrong one, *E* said so, and let the clock run until she found the right one.

When the right pattern had been found and the clock switched off, there was a pause of about 20 sec. during which *S* might rearrange the patterns on the board. She was warned before the next pattern appeared on the screen.⁵

After about half the patterns had been presented, *E* called a temporary halt, and explained that he wanted to ask some questions about the arrangement *S* had so far produced on the board. The questions were of the form: "Suppose I were to interchange these two patterns, would this make any difference to how easily you find the patterns?" "What about if I changed the order of these six?" etc. Before this procedure was started, it was made clear to *S* that the questions were all neutral. Some of the changes mentioned might improve her arrangement, whereas others might not. All that we wanted to know was: would the suggested change make any difference to *her*. None of the suggested changes were actually made, though *S* was free to make them later if she wished; in practice, the questions never led to radical reorganization.

The reason for this question period was twofold. First, we wished *S* to clean up any "loose ends" in her arrangement and consolidate the ideas she was developing. For our purposes we naturally wanted as explicit a statement as possible about *S*'s perception, and therefore wanted the arrangement to be as unambiguous as possible. Secondly, the questions helped to clarify for *E* what *S* was doing. Often a large block of patterns which were grouped together (say 10 or 12 of them) would be thought of by *S* as made up of several subgroups. Although there was no physical separation between the subgroups, it

⁵One or two *S*s began to separate the patterns which had appeared already, from those which had not. These *S*s were reminded that any pattern might appear more than once, and were asked to make an arrangement that did not discriminate between patterns that had been shown and those which had not. In fact, since there was just one slide for each pattern, and the slides came in a fixed order, *S* saw each pattern just once. However, as far as we know, no *S* ever realized this.

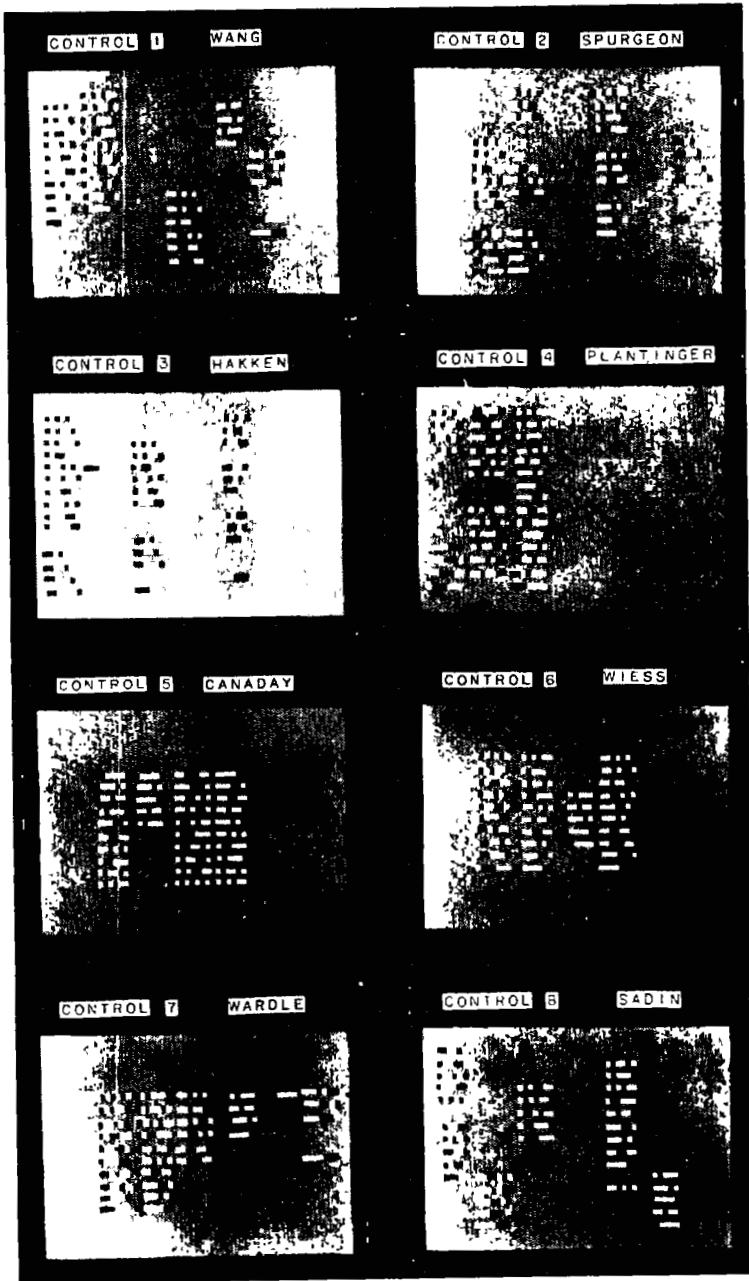


FIG. 3. Control group (half)

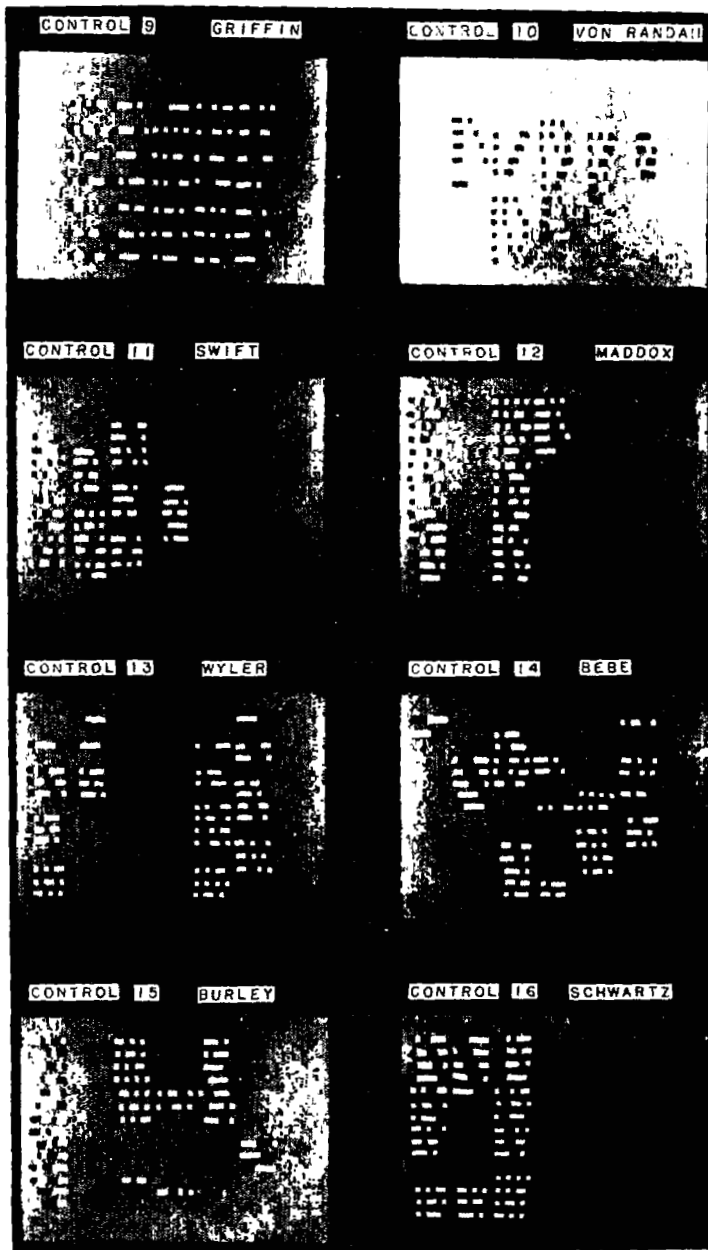


FIG. 3. Control group (half)

was possible to detect the presence of these divisions by means of the questions. If *S* thought of two adjacent patterns as being in the *same* subgroup she didn't mind interchanging them; if she thought of them as being in different subgroups, however, she would not allow them to be interchanged.

After this question period, the test went on as before. When the last slide had been presented, *S* was told not to make any more changes and was asked to give a verbal account of what she had tried to do, so that we could be quite sure we understood the grouping of patterns on the board correctly.

All *Ss* took this test. However, for the 16 *Ss* who form the control group, this was their first and only exposure to the material. The arrangements produced by the control group are shown in Fig. 3.⁴

Experience 1: Random Search Experience

The purpose of this experience was to investigate the effects on perception of searching in totally unstructured and unstructurable situations. The experimental procedure was the same as that described for the test, except that in this case *S* was not allowed to move the patterns about on the board at all. She thus had to find each pattern, as it was presented, by searching for it in the fixed random-like array. *S* searched for each of the 35 patterns, in each of two arrays. These two random-like arrays, Nos. 2 and 3 in Fig. 2, were both different from the one which starts off the test situation but they were constructed in the same way, and were intended to be as different from one another as possible to avoid any position learning effects. The patterns appeared on the screen in the same order as in the test. *S* was given the following instructions:

There are 35 patterns here, all different. I am going to show one of these patterns to you at a time, there on the screen, and as soon as I show it to you I want you to find the pattern among those on the board.

All the patterns I shall show you will be on the board; there are no tricks. I may show you the same pattern more than once, or even several times.

Every time I put a new one on the screen, I shall start this clock, and I want you to find the corresponding one on your board as fast as you can. It must be the same way up as the one on the screen. When you find it, point to it. When you point to the right one, I shall stop the clock. You may not move the patterns on the board. After this experience, which lasted about $\frac{1}{4}$ hr. for each array (a total of $\frac{1}{2}$ hr.), *S* was given the test; the 8 *Ss* produced the arrangements illustrated in Fig. 4.

Experience 2: Tachistoscopic Experience

The purpose of this task was to give *S* experience with the patterns, but under conditions in which they were available to her as input for such a short time that she had to work hard to identify them. *S* was given the following instructions to read.

⁴It should be noted that the numbering of *Ss*' arrangements in the figures is for ease of reference, and is not the order in which *Ss* were run.

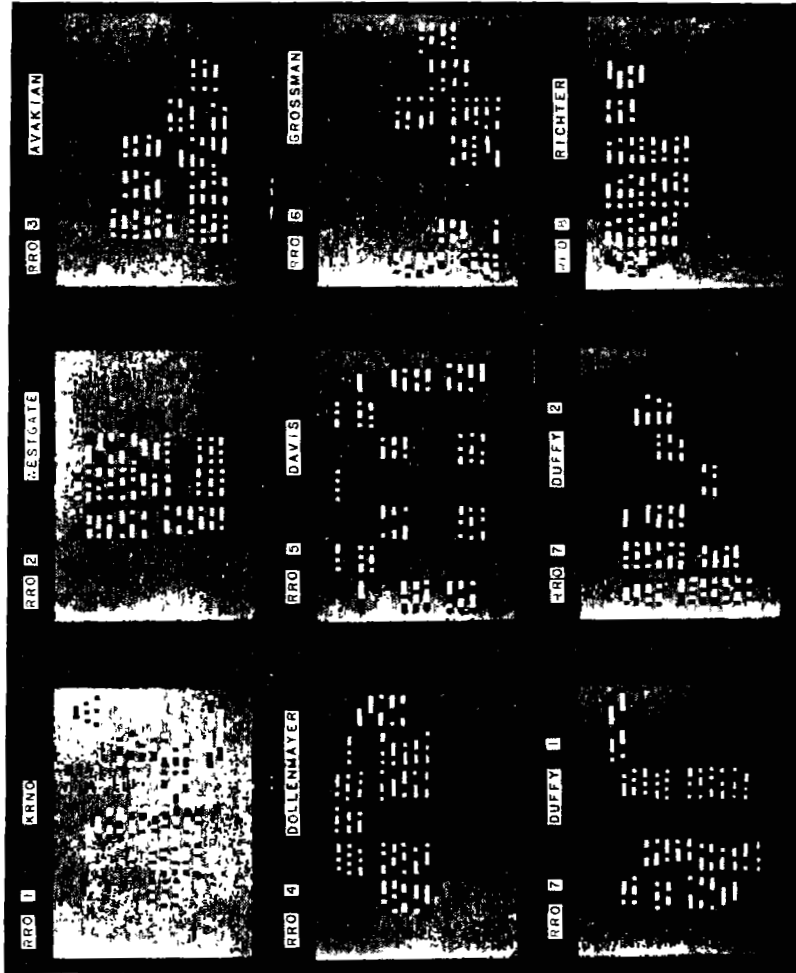


FIG. 4. Random Search group



FIG. 5a. Tachistoscope group

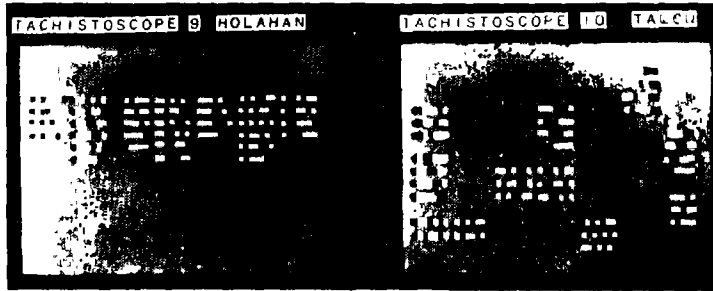


FIG. 5b. Tachistoscope group

This is an experiment in perception. In the first part of the experiment, I shall flash patterns on the screen in front of you, one at a time. They will appear inside this frame. The patterns are very simple. At the end of each presentation, there will be a short, very bright flash.

What I want you to do is to fill in the pencilled outline on the strip of paper, until it looks like the pattern you have just seen. Try to work fast, but draw them accurately, so that you would be able to select the pattern from some others like it, if you were asked to later. When you have finished, fold it under so that you cannot see it, and tell me, so that I can flash the next pattern. Are there any questions?

S sat in front of the screen, and the patterns were projected on it one at a time, in the usual order. They appeared inside an outline pencilled on the glass so that S knew where to look. The exposure was controlled by a photographic shutter, and lasted about 20 msec. As the shutter closed, it fired an electronic photoflash which was directed at the fixation outline from behind the screen. This served to kill the after-image, thus ensuring that we had control of the effective duration of the image.⁷

S was given a strip of paper on which appeared pencilled outlines of about the same size as patterns appearing on the screen. Her task was to reproduce the pattern she had just seen, in pencil, within the outline. After each drawing was completed, it was folded under so that S did not have any opportunity to look at her previous efforts. After the 35 slides had all been presented in the usual order (this took about ½ hr.), Ss were given the test. The 10 Ss produced the arrangements illustrated in Fig. 5.

Experience 3: Play Experience

The purpose of this procedure was to investigate the effects of "creative" play on S's perception. S sat down, and was given the following instructions: Here are some patterns. I want you to play with them and get to know them.

⁷Compare work of Baxt (1871) and Sperling (1960 a, b). A piece of plain glass extracted some light from the projector beyond the shutter, and this was used, by means of a photosensitive resistance element and two Mercury relays, to fire the flash gun when the shutter closed.

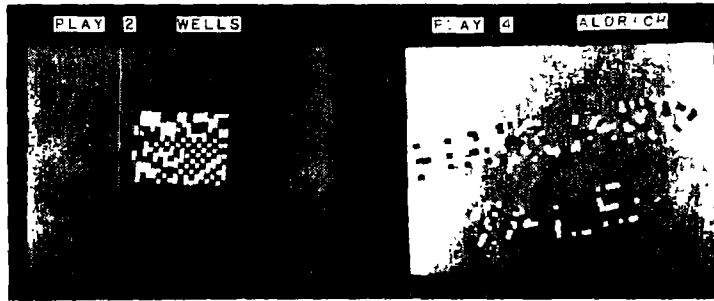


FIG. 6. Play examples

Then, as *S* began to make a pattern of some sort with the pieces, as she inevitably did, she was told further:

If you want to make a larger pattern with them, don't feel constrained; make any kind of pattern you feel like. Imagine you're an artist and "doodle" with them.

When she had finished, we photographed the arrangement, and then scrambled the pattern up on the board. She was then told:

Now please start all over again. This time make a pattern as different from the first as you can.

This arrangement was photographed and the patterns scrambled. The third time she was told:

Now please start again, and make a pattern as different as possible from the other two. Please give a good deal of thought to each individual pattern this time. Before you decide just where to put it, in relation to the other patterns, look at it very carefully, and try to decide in your own mind just where it 'belongs.'

This arrangement also was photographed. After this play period which lasted about $\frac{1}{2}$ hr., *S* was given the test. Two examples of play arrangements are illustrated in Fig. 6. The arrangements that the 12 *Ss* produced in the test period are illustrated in Fig. 7.

RESULTS

Tabulation and Analysis of Data

We first describe our method of classifying the test arrangements. We wish to discriminate between perceivers who see a black and white strip as a sequence, and those who see it as a whole. We do this by examining the groups of patterns which *S* creates. It is usually easy to identify these groups, because they occur in different columns on the board, or because they are separated from one another by empty cells. However, essential supplementary information about the groups *S* considered herself to be using, was obtained during the question time in the middle of the test (cf. Test Procedure above). To help the

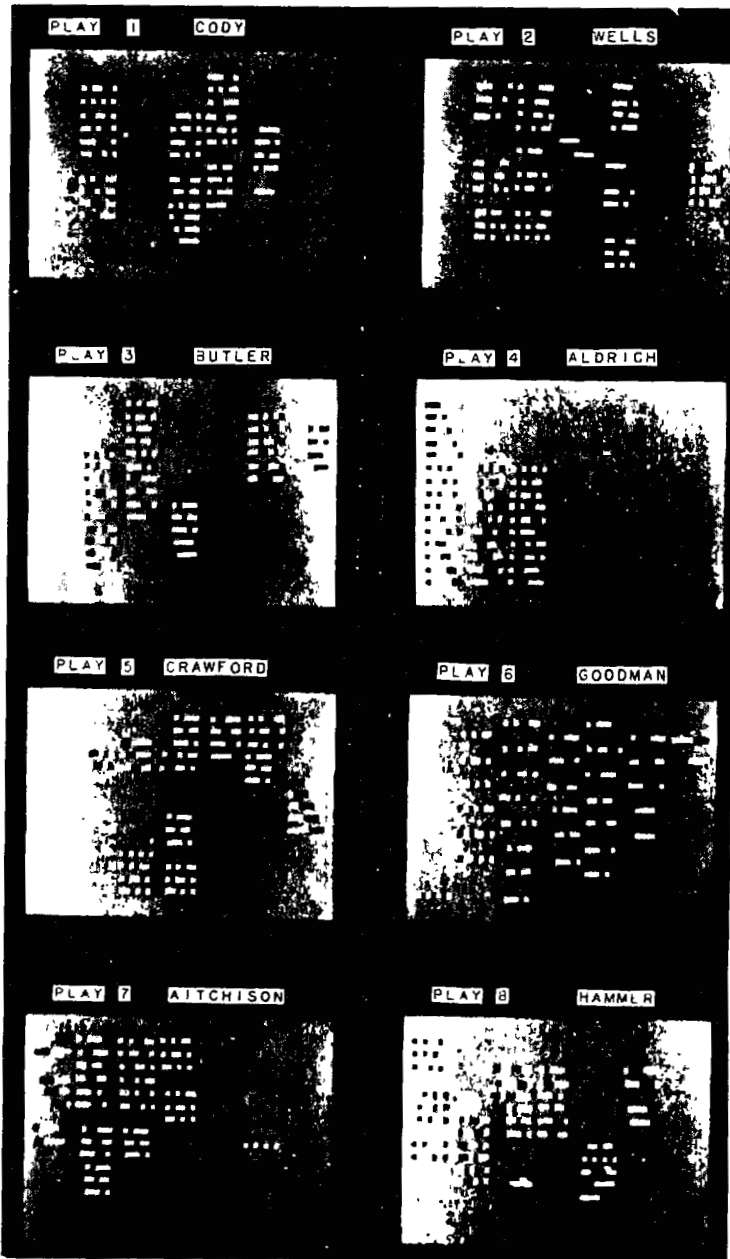


FIG. 7a. Play group

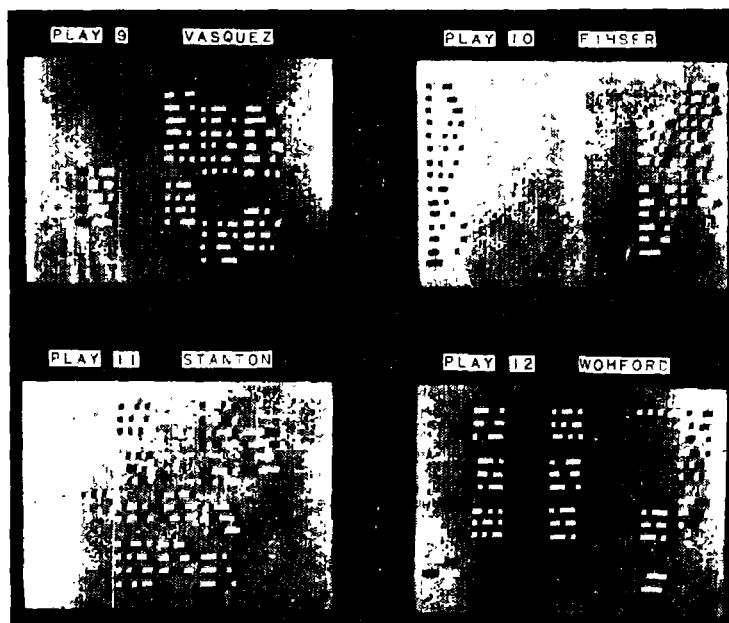


FIG. 7b. Play group

reader understand how the distinction between sequential and figural is based on the groups, we have constructed two arrangements as examples, one based on sequential groups, the other on figural groups (Fig. 1).

A glance at Fig. 1 will make it clear that in the sequential arrangement, those patterns which start with black are separated from the ones which start with white. Within the "black" group, the patterns that start with a single black square are separated from those which start with two black squares. Within each of these subgroups the patterns are further grouped according to the length of the second block, and so on. This leads to an arrangement essentially isomorphic with the binary numbers. Search for a pattern is based on a left-right reading of the color and size of the individual blocks within the pattern. *The pattern is seen as a sequence of units.*

In the figural arrangement the sequential position of blocks in the patterns is less important than the type of structure each pattern exhibits as a whole. Each pattern is seen as black figure on white ground, so that those patterns that have the same figure are grouped together (whichever way around the figure is, and wherever it appears in the pattern). Within these groups, patterns are further grouped according to the position of the figure in the ground. *The pattern is seen as a single unit.*

The two arrangements shown in Fig. 1 are easy to classify as sequential and figural, respectively. They are unambiguous. Many of the arrangements produced by *Ss* were equally unambiguous. However, there were some in which both sequential and figural groups of patterns occurred, and these introduce a need for an objective method of classification. Instead of trying to give a general rule for what constitutes a figural group, it is easier to define the concept extensionally, by listing the following possible groups. (Notice that in every case the property must hold for *all* patterns in the group.)

1. Every pattern in the group contains a single long black block, and no other black. (See, for example, Fig. 3, Control 16.)
2. Every pattern in the group contains three black squares (e.g., Fig. 4, Random Search 4).
3. Every pattern in the group contains two black blocks, one short and one long; and some of these patterns are related systematically to their mirror-images, or to the patterns that contain mirror-images of their figures, so we are sure that it was the figure that was important, not the sequence (e.g., Fig. 4, Random Search 5).
4. Every pattern in the group has the same number of black-white boundaries (e.g., Fig. 7, Play 6).
5. Every pattern in the group is symmetrical (e.g., Fig. 4, Random Search 1).
6. Every pattern in the group has its white broken into the same number and size of pieces (e.g., Fig. 7, Play 11).
7. For every pattern in the group, the mirror-image is also in the group (e.g., Fig. 5, Tachistoscope 10).
8. For every pattern in the group, the pattern obtained by reversing just the black figure is also in the group (e.g., most of the groups in Fig. 3, Control 14, but especially the group of four in the middle of the right hand column).
9. All of the patterns in the group are "simple" i.e., contain three or less blocks, or are symmetrical, e.g., Fig. 4, Random Search 8.

We classify a group of patterns as figural if it meets one of the above nine criteria; otherwise we classify it as sequential. The number of groups of each kind in the different arrangements, are shown in Table 1.

We classify each arrangement as a whole, as figural, sequential, or intermediate, according to its constituent groups, as follows. An arrangement is classified as figural if all the groups in it, with at most one exception, are figural. An arrangement is classified as sequential if all the groups in it, with at most one exception, are sequential. If an arrangement contains more than one figural group, and more than one sequential group, it is classified as intermediate.

We give a summary of Table 1, and the probabilities of these results, in Table 2 below. The probabilities given in the last column permit us to reject the null hypothesis that all groups are drawn from the same population.⁸

⁸A modified Fisher exact probability test was used; see Appendix for details.

TABLE 1
DATA FOR EACH GROUP AND INDIVIDUAL

Control group (Fig. 3)	
<i>S</i> identification	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Total groups in <i>S</i> 's arrangement	8 9 8 7 6 6 7 7 1 8 8 7 10 10 7 6
Number of figural groups	1 0 0 6 1 0 0 0 0 5 1 0 9 9 7 6
Number of sequential groups	7 9 8 1 5 6 7 7 1 3 7 7 1 1 0 0
Classification†	S S S F S S S S S I S S F F F F
Random Search Group (Experience 1) (Fig. 4)	
<i>S</i> identification	1 2 3 4 5 6 7a* 7b* 8
Total groups in <i>S</i> 's arrangement	7 7 9 6 11 10 8 8 6
Number of figural groups	7 7 9 6 11 2 1 7 2
Number of sequential groups	0 0 0 0 0 8 7 1 4
Classification	F F F F F I I I
Tachistoscope groups (Experience 2) (Fig. 5)	
<i>S</i> identification	1 2 3 4 5 6 7 8 9 10
Total groups in <i>S</i> 's arrangement	8 7 8 10 6 8 8 10 7 9
Number of figural groups	7 7 0 6 5 4 1 10 0 8
Number of sequential groups	1 0 8 4 1 4 7 0 7 1
Classification	F F S I F I S F S F
Play group (Experience 3) (Fig. 7)	
<i>S</i> identification	1 2 3 4 5 6 7 8 9 10 11 12
Total groups in <i>S</i> 's arrangement	7 11 8 8 9 6 9 9 7 4 11 11
Number of figural groups	1 1 4 6 9 6 9 9 0 0 11 11
Number of sequential groups	6 10 4 2 0 0 0 0 7 4 0 0
Classification	S S I I F F F F S S F F

†Classification as Sequential (S), Intermediate (I), Figural (F).

*Subject 7 stopped in the middle of the test, to say: "There are two ways of doing this," and demonstrated by making both arrangements 7a and 7b. We have illustrated both arrangements, but have classified them as a single intermediate.

TABLE 2
PROBABILITIES ASSOCIATED WITH DATA OF TABLE 1

	No. of <i>S</i> s classified as:			<i>p</i> *
	Sequent.	Inter.	Figural	
Control group	10	1	5	
Random search	0	3	5	0.006 < <i>P</i> < 0.018
Tachistoscope	3	2	5	0.11 < <i>P</i> < 0.14
Play	4	2	6	0.11 < <i>P</i> < 0.13
Pooled total	7	7	16	0.010 < <i>P</i> < 0.014

*This group was drawn from the same population as the controls.

Findings

The random search procedure has a strong effect on the way people see. About three quarters of the *S*s who would (judging by the control group) have seen the patterns as sequences, after exposure to the random search experience instead see them figurally.

The Play experience and the Tachistoscopic experience have a smaller

effect, the effect they do have is in the same direction as the effect of Random Search. They tend to make *Ss* see figurally.

DISCUSSION

The most striking aspect of these experiments is the definite direction of the perceptual changes which occur. The three kinds of experience, "search on a random-like array," "exposure to tachistoscopic presentation," and "play," were chosen to be free from any explicit bias toward figural perception. What is more, sequential perception, according to remarks made by *Ss* during the experiment, is a very neat, systematic way of dealing with the patterns. Since all three kinds of experience induce a tendency in *Ss* to abandon this mode of perception and to learn to see the patterns as wholes, rather than as sequences, we must infer that the figural mode is in some way more efficient than the sequential.

Why does Random Search induce this change? The first and most obvious hypothesis is that mere exposure to the patterns tends to make *S* see them figurally. However, the weakness of the Play experiment makes it clear that this is not the main part of the effect. In all three procedures *Ss* dealt with the patterns for about the same length of time ($\frac{1}{2}$ hr.), yet the effect of Play is noticeably weaker than that of Random Search.

A second hypothesis that suggests itself, is that perceptual 'hard work' favors the most efficient mode of perception, and so induces a change towards figural (Allan, 1961). This would explain why Play, which is 'easy' perceptual work, has little effect and why Random Search has a strong effect. *S* searching on the random array has to work extremely hard. For each stimulus presented, *S* has to look at several patterns, some perhaps more than once, rejecting each one until she finds the one presented. Under time pressure, *S* is forced to work as hard as, and for much longer than, even in the Tachistoscope presentation. Yet this is still not an entirely satisfactory explanation for the success of Random Search. If the "hard work" hypothesis were entirely correct, we should expect Tachistoscope procedure, which also makes *S* work hard perceptually, to have a stronger effect than Play. Yet it does not. Apparently there must be some further reason for the effect of Random Search.

We make the following suggestion. When faced with a novel array, the perceiver tries to organize it. He looks for groupings of patterns within the array. In the case of a random array, he is prevented by the nature of the array, from establishing any groups on the basis of adjacency; so instead he tries to establish groups by the geometry of their location. For example, take the two patterns which contain just two blocks each, one black and one white. These two patterns, which are mirror images of one another, stand out very strongly as a pair; one remembers their position, not as individuals, but as a pair. In Random Array 1, they occupy the two cells; row 5 column 3, and row 7 column 5. It is these two cells together, and their relative positions, that one remembers;

not which of the cells contains which pattern. Another set of patterns that are remembered as a set rather than individually, are the three in which there is a regular alternation of black and white squares. In Random Array 1 they occupy the three cells: row 4 column 1, row 3 column 3, and row 5 column 4. Random Search Ss quickly became aware of these groupings, and of others," but the patterns seen to form these spatial groupings were always figurally similar, and never sequentially similar. Why was it not also possible for S to see patterns that were sequentially similar in such geometrically extended groups?

The reason is very likely this. Any search task demands that S's mode of perception for the whole board be the same as her mode of perception for the individual patterns. In her own arrangement, S was free to choose whatever mode or rule she pleased. But it is not possible to invent a rule which makes sequential perception of the arrangement an efficient search procedure in the random array: it takes too long. In order to save time, S is forced to take a figural approach to the random array as a whole. She cannot integrate this approach with a sequential approach to the individual patterns. From a sequential point of view, the salient part of a pattern is its left-hand end. Yet in the random array there is nothing to make the left-hand ends of the patterns stand out from the right-hand ends. S is forced by the disorderliness of the random array to look at the array as a whole, and therefore has to find, in the patterns, recognition units whose saliency is not destroyed by this holistic approach. Only the black figures have this property. Only a figural perceiver sees these as units.

The Random Search experience makes S see the patterns as whole, or as figures, because she has to integrate her perception of individual patterns with her perception of the random array as a whole.

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*We know this from Ss' verbal reports and from watching their perceptual confusions.

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Accepted June 3, 1964.

STATISTICAL APPENDIX

The Fisher exact probability test is used to analyse frequency data that can be represented in a two-by-two contingency table, when the expected frequencies are too small to allow the use of a chi-square test. The Fisher test asks the question: what proportion of all possible frequency distributions having the same marginal totals are as extreme as, or more extreme than, the observed distribution? The same question can be asked when the data fall into more than four cells, but a problem arises. It is not obvious, as it is in a two-by-two array, which distributions *are* more extreme than the observed distribution.

The data from any one of the present series of experiments fall into a two-by-three array as follows:

	Sequent.	Interm.	Figural	
Control Group	A	B	C	A+B+C
Experimental Group	D	E	F	D+E+F
	A+D	B+E	C+F	N = A+B+C+D+E+F

The probability that *N* objects are distributed in this way, given the marginal totals, is given by the hypergeometric function:

$$P = \frac{\binom{A+D}{A} \binom{B+E}{B} \binom{C+F}{C}}{\binom{N}{A+B+C}} = \frac{(A+D)!(B+E)!(C+F)!(A+B+C)!(D+E+F)!}{N! A! B! C! D! E! F!}$$

Since B and E refer to an intermediate category, the extremeness of a distribution is determined solely by the relationships between A, F, C, D. With the marginal totals fixed, the values of A and F completely determine the values of C, D, B, E, so that we may distinguish only three cases: those in which A+F is larger than in the observed distribution, those in which it is smaller, and those in which it is the same. Any distribution in which A+F is larger (with C+D consequently smaller) is clearly more extreme than the observed distribution. Any distribution in which A+F is smaller (with C+D consequently larger) is, at least for the present range of frequencies, less extreme than the observed distribution. A distribution in which A+F (and consequently C+D also) is the same as in the observed distribution, is neither more nor less extreme. It is not clear which of these middle cases should be included when we compute the probability. We have therefore computed an upper and a lower bound on the probability, the upper bound including all the middle cases, the lower bound including only the one middle case corresponding to the observed frequencies, and excluding the others.