THIRD SERIES VOL 66 NUMBER 12 OCTOBER 1959

THE JOURNAL OF THE ROYAL INSTITUTE OF BRITISH ARCHITECTS

66 PORTLAND PLACE LONDON W1 · THREE SHILLINGS AND SIXPENCE

Kings Weston. Architect: Sir John Vanbrugh *Eric de Maré*

'La, tout n'est qu'ordre et beauté, Luxe, calme et volupté'

Perception and Modular Co-ordination

by Christopher Alexander

THE USE of geometrical or arithmetical systems to provide internal order in buildings was favoured by the Greeks, by the Renaissance Italians, and even, if we are to believe Mr. Lesser, by the Gothic masters. The conviction that an order of this kind plays a major part in producing the experience we call beauty, is a deep-rooted one.(1)
The conviction has, however, never been dissociated entirely from mysticism and tends either to be rejected altogether by 'reasonable' people for its air of black magic, or to be countenanced for quite the wrong reasons by the mystically inclined.

It will turn out, I hope, that it is possible to accept the conviction with some confidence, but for good reasons instead of bad. Our inquiry will of necessity be humble—since work is almost never done in this field, and there are no secure foundations. But the problem can be discussed, if we think clearly and get rid of the confusion that besets it.
We shall deal with it as follows:

1. We shall consider this generally accepted view, that order is visually desirable; discuss the view in the light of recent theories of perception, and indicate how the view might, in fact, be justified.
2. We shall examine the cult of the golden section; and show that the claims made for it are in large part exaggerated—that the order this system *does* provide can be provided just as well by countless other systems which are only less well known because no attempt has been made to mysticise them, to make religions of them.
3. We shall see that the golden section and other current module-centred systems are particular cases of a *general* system—which may indeed be connected with the facts of perception as we understand them. We shall develop this general system, and make it workable.
4. Finally we shall see that this general system is, as it happens, closely connected with modular co-ordination—that technology, in fact, is developing a system that is intimately related to ours. We shall discuss ways in which these two systems, one the result of technological discipline, the other of a theory of visual order based on perception, can be used conjunctively.

Visual order is taken to be some quality in the patterns that make up a building. (We see patterns wherever the components of which the building is made are visually distinct.)
First, then, we are interested in the part played by such visual order in the experience of someone who finds a building beautiful. Whatever we find, it is never going to be responsible for a great deal of the experience—structure, materials, space enclosed, are all much more important. But it does play its part. And it is this part that we are interested in.

Before we can go any further we must be entirely clear about our use of the word 'order'.
When one of Mondrian's paintings, for example, is said to possess a high degree of order it is not at all clear what is meant. 'Order' is used much as 'excellence' might be—it tells us hardly more than that the painting is a good one of a certain sort. (This is not to say that Mondrian's paintings are *not* ordered. Only that their order is too elusive for us to understand it; we are too stupid yet, perhaps, to see just what it is.)
In this discussion we shall only use 'order' where we can give the word operational definition, where we can *point to* the order, where we can say '*This* is what makes it ordered'.

The obvious meanings of 'order' refer to some kind of simplicity, symmetry perhaps, lack of complication, lack of distraction. It is not difficult to decide on an operational criterion according to which we can make up our minds whether to call an object ordered or not. Suppose we decide on some such criterion. Is it then true to say that those objects that are 'ordered' are easier on the eye than those that are not 'ordered'? And if we *assume* this to be the case, is there anything in the mechanics of perception that justifies the assumption? Or, to put the question another way, is there anything about objects that are ordered that might lead us to expect them to be visually satisfactory?
We must admit right away that there is no conclusive evidence one way or the other. But there are some indications.
All three theories of perception that we shall examine (and they are among the most important current accounts), have this in common: they maintain that seeing involves an effort, and that the perceptual mechanism works in such a way as to minimise this effort.
It is no great step from here to the answer we want. For if the object being seen is simple (or 'ordered'), the effort that the mechanism needs to make is particularly small—and the situation is, from the point of view of the lazy brain, *satisfactory*.

The gestalt account of perception is ruled by the principle of isomorphism. (2)
The theory suggests that whenever an object is perceived, its form re-occurs somehow in the nervous system. That is to say, the form of the physiological configuration in the brain is isomorphic to (structurally analogous to) the form of the object.
This explains, for instance, why ambiguous figures are always seen in the simplest possible way—the brain seeks to organise itself in the least complicated fashion, so a trapezium is not seen as a trapezium in the frontal plane, but rather as a rectangle in perspective.
And it is quite clear that in terms of this account an 'ordered' object will allow the brain more rest than a complicated one. If the object itself is simple, so will the situation in the brain be—and the state of affairs in the perceptual mechanism will be satisfactory.

In Hebb's account of perception the ruling idea is one of aggregation.(3) There is no overall or field theory, but instead, the suggestion that we see figures as the result of a complicated learning process which goes on through cell assembly in the visual cortex.
The basic assemblies are formed very early in our seeing life, and later we combine these basic ones in order to see more complicated patterns, possibly adding still further cells.
Again, it is the simplicity of the figure that gives the brain an easy job. If the brain can use a particularly small set of basic assemblies, and does not need to add further cells to them, there will be a low level of activity in the visual cortex—the figure will be easy on the eye.

The information theory account, given by Attneave, is most interesting.(4) Most patterns are highly redundant, in respect of the information they give. Thus, if we describe the visual field in terms of a minute grid, each square of which is monochromatic—like the grain of a photograph—and if we tell somebody the colour and tone of these tiny squares one at a time, not all our information will surprise him. In the case of a simple pattern he will anticipate the colour and value of most of the squares we come to (as soon as he realises what pattern it is that we are describing, he will know the values of *all* the remaining squares), while with complicated patterns he may be kept guessing most of the time. The complicated pattern is said to contain more information than the other—in fact, the number of errors he makes in predicting future squares is an index of the amount of information carried by the pattern. Attneave implies that each error costs the observer an effort, so that simple patterns, which carry less information than complicated ones, cost him a smaller effort than the complicated ones.
Again then, a pattern that is 'ordered', since it costs the brain less effort than others, is more satisfactory to the eye.

Each one of these tentative 'explanations' of the fact that ordered things are more apt to please the eye than others depends on the fact that the brain (or eye, if you prefer) is lazy.

And, of course, this view is not unarguable. There are ways in which just the contrary seems to be true—where it is suggested that the eye has to be kept *busy*.[5]

Goodyear's account of the refinements of Greek and Egyptian architecture suggests that the architects of these periods used entasis and similar devices, not to correct optical illusions, as is so often thought, but to achieve something positive.

To see lines that are curved as straight, or unequal intervals as equal, the brain has to make compensatory efforts, and strain itself far more than when it sees *equal* intervals as equals. It may be that this very effort is pleasurable, and that the refinements titivate the eye and keep it happy, so to speak.

And there is certainly something to be said for this idea.

But it does not exclude the other, and much older view, that simplicity rests the eye and is therefore beautiful.

Many, many societies have held this view—and of them, many made it into some kind of system. Both in Japan and India, for example, it has long been regarded as important.[6]

In Europe the idea has been several times attached to the whole of religious thought. The Greeks, the Gothic builders, and the Renaissance scholars, all valued some such theory (though possibly it was no more than a desire to simplify measurement that made it attractive in the Middle Ages); and now, at the moment when hope of understanding visual aesthetics is just appearing, the architectural world has been inundated by further mysterious writing on the golden section and geometry. Instead of trying to account for the effect of order in a way appropriate to our time, the majority of writers have returned to an almost primitive acceptance of magic and ritual.

Those that have promoted this return to the golden section are, unfortunately, often distinguished enough in their own fields to make it inconceivable that they should be mistaken. The absence of even the beginnings of careful analysis in this subject is attributed, not to the inability of its exponents, but to the nature of the subject itself.

Yet we have only to examine the work in detail to see how flimsy its foundations are. The failure of writers to appreciate the true reason for the visual efficacy of the golden section has led them to shelter in a maze of obscurity. It is clear from their whole approach to the subject, and from the vagueness of their so-called proofs and demonstrations that they themselves are quite uncertain—that they are unable to account adequately for the facts.

Let us examine some of these 'proofs'.

First of all, throughout the writings we are concerned with, there seems to be deliberate intention to hoodwink the reader. While this *need* not have any bearing on the value of the idea itself, we cannot help wondering why it is so common; whether it is not, in fact, because the only way to prove things that are incorrect *is* by false argument. Or perhaps it is simply that the writers are too ignorant to know what they are doing. Le Corbusier, for instance, reverently reproduces facsimiles of two pages of arithmetic a mathematician did for him.[7] The arithmetic involved could have been done by many schoolboys, and to suggest that it is difficult by showing readers the original manuscript is sheer deceit.

Similarly, Matila Ghyka glibly invokes Ockham's razor at a point where the razor principle has no application and does not in any way help the idea which he intends it to support.[8]

Its use is a pretence.

Another attempt to hide behind mathematics and impressive words is made by Jay Hambidge when he explains his devotion to the golden mean by saying that it is a dynamic ratio rather than a static one.[9] He means, it later turns out, that the dynamic ϕ is an irrational number like $\sqrt{2}$ or π, while the static numbers are integral fractions.[10] Now, although there is no harm in putting this forward as new terminology, it is senseless to describe the difference as significant when discussing the way people react to patterns.

The irrational numbers make no sense as physical lengths. Physical lengths, which are by definition commensurable, must not be muddled with numbers that are entirely abstract.

Think of this another way: as regards physical lengths, since there can be no irrational ones, it makes no sense to distinguish between 'static' and 'dynamic' ratios. There is NO way in which this might be done, since there is always a rational number as close as you like to any irrational one.

Even disregarding this logical point (vital though it is), the business is *still* absurd. The limitations of visual acuity make nonsense of it. In general, people cannot distinguish between two rectangles whose height-breadth ratios are 6 per cent apart. That is to say, if we have two rectangles, one a by b, the other a by $1 \cdot 06b$, observers cannot tell one from the other.[11]

Thus, though observers can distinguish between a square and a ϕ rectangle, and even between a $1:1 \cdot 5$ rectangle and a ϕ rectangle, the $1:1 \cdot 66$ rectangle and the ϕ rectangle are visually the same. For an exceptionally acute observer we might need to make the difference finer—but the point remains. When we see, we see, not a rectangle with the mathematical properties that $(1-\sqrt{5})/2$ has, but a shape whose sides are in a ratio somewhere between $1 \cdot 6$ and $1 \cdot 65$. (Fig. 1.)

To make distinctions between rectangles whose ratios are static and rectangles whose ratios are dynamic, is to befuddle ourselves.

Closely related to the facts of visual acuity is another favourite device of golden sectionists. We are confronted with (and convinced by) analyses consisting of hundreds of lines ruled across plans and elevations. (Fig. 2.)

Now both the lines of the analsyis, and the beadings, mouldings, frames, etc., of the object itself, are so thick (in relation to the object's overall dimensions), and the intersections can be so variously made, that any consequent deductions and results are valueless. (Fig. 3.)

A perfect example of the way in which line thickness variations can be used is the famous 'paradox' of the rectangle and the square. The first, consisting of 65 unit squares, appears to be made of the same elements as the second, which contains only 64. In fact the lost unit is taken up by the area of a line—but imperceptibly, because the lines are all thick enough to cover the deceit. (Fig. 4.)

But the fact that line analyses prove nothing does not daunt the mystic. He has far more impressive evidence for the uniqueness and special qualities of the golden mean.[12]

The occurrence of the golden mean in nature. The Fibonacci series and the geometrical figures associated with it, pervade, so it would seem, the world of natural forms. And this is taken to be a clear indication of the mystical qualities of ϕ.

It is true that we find pentagons, five-petalled flowers, equiangular spirals, serial arrangements of leaves on branches. (Fig. 5.)

But all these patterns are governed by the way in which they have been made. They are not the result of nature striving for some high ideal—simply the outcome of a certain kind of growth.[13]

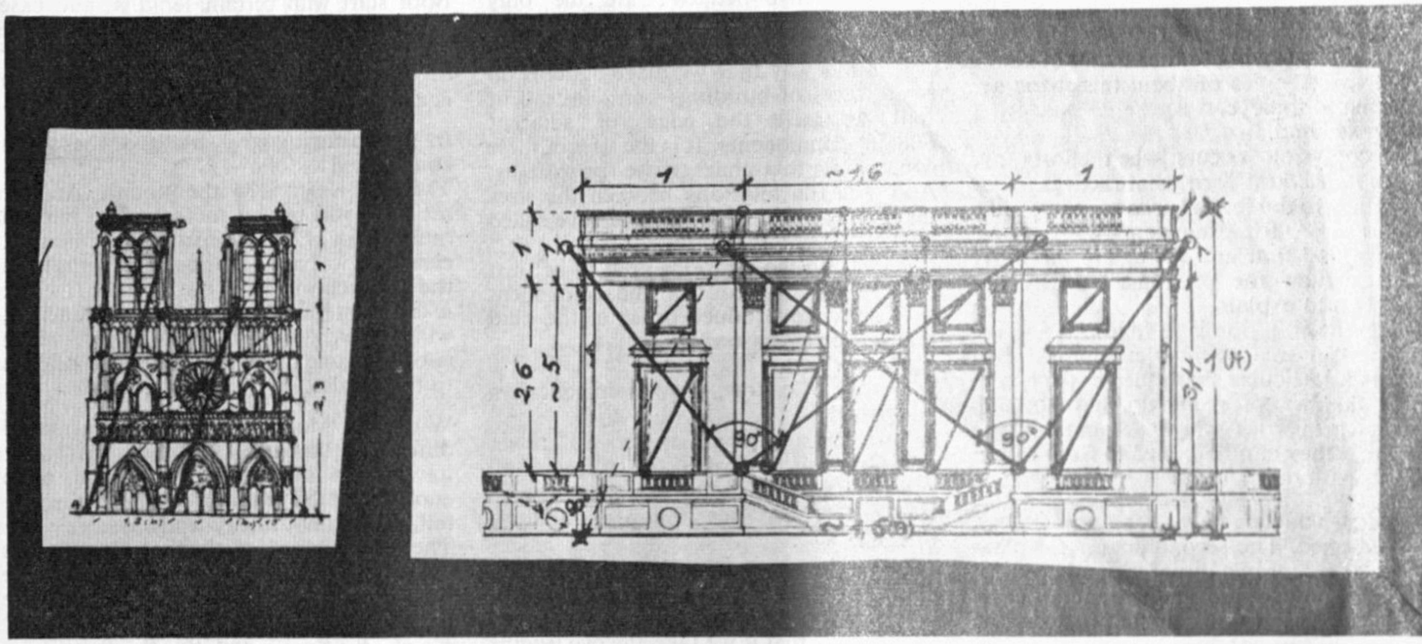
No more remarkable than the fact that the two halves of a dicotyledonous seedling are the same—it is a result of the way these plants develop. This was made clear as early as 1872, when it was pointed out that unless we can give literal sense to the idea of a plant 'aiming at something', the idea is absurd.[14]

Yet almost 100 years afterwards we still find people writing as though nature uses the golden section in order to be harmonious.[15] The numbers actually found in nature (ratios like 5/8, 3/5, 2/3, 1/2), are not at all close to ϕ, while those that *are* close to ϕ (3/21, 89/144, and so on) are never found.

ϕ itself plays no part in natural growth; but the first few members of the Fibonacci series, and the structure of this series, do picture the serial growth of certain forms. This is not mysterious in any way. And what is more, there is no reason (except a Platonic one) to consider the series particularly significant. The fact that it is associated with a certain principle of growth says nothing about the visual effects of the results. Nor should it dictate anything to us as formbuilders—unless the ways in

Fig. 1. We can barely distinguish between these two rectangles, one 1: 1·6, the other 1: 1·65. →

Fig. 2 below. From *The geometry of art and life*, by Matila Ghyka.



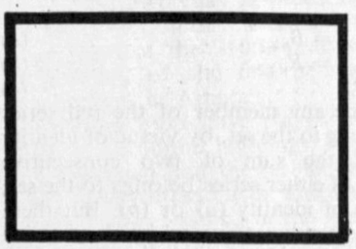


Fig. 3. The inner rectangle is 1: 1·57.
The outer rectangle is 1:1·51.

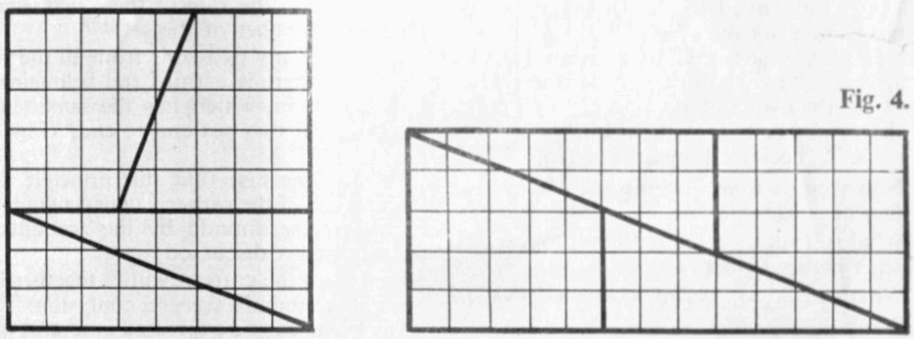


Fig. 4.

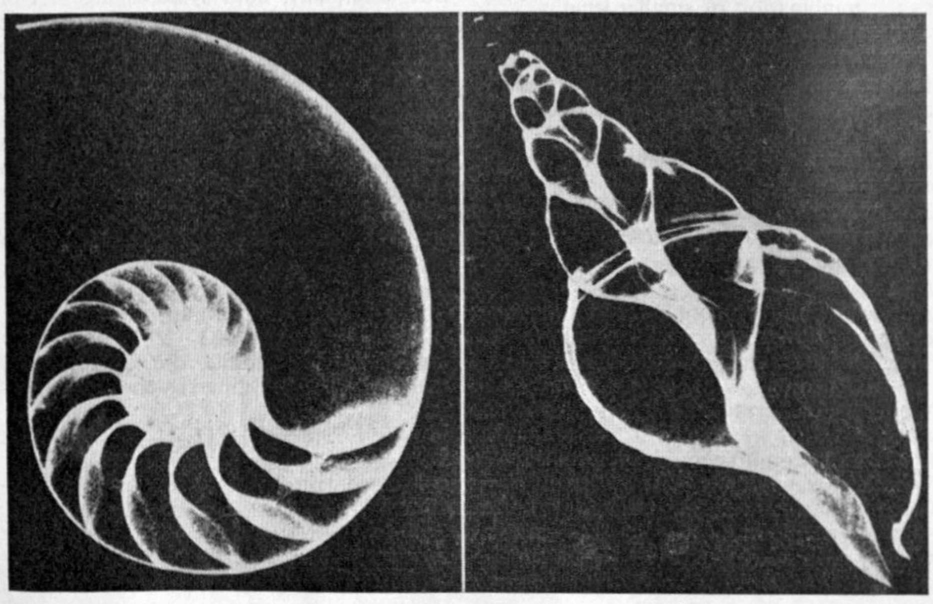


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Fig. 5. From *The geometry of art and life*, by Matila Ghyka.

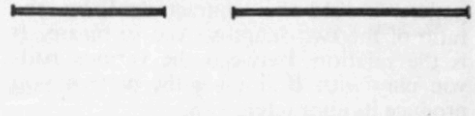
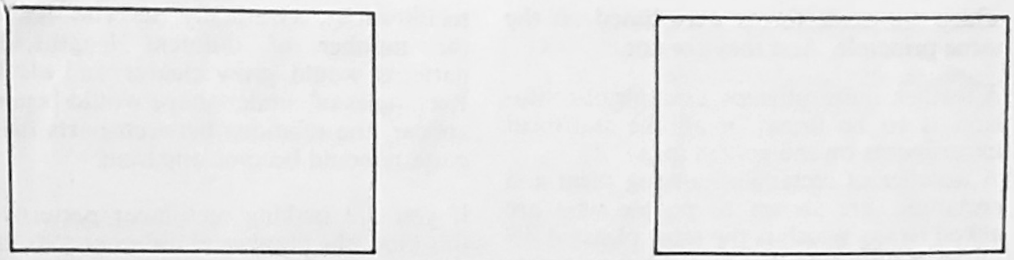


Fig. 6. Given any two rods from the collection, their sum also belongs to the collection.

Fig. 1. We can barely distinguish between these two rectangles, one 1: 1·6, the other 1: 1·65. →



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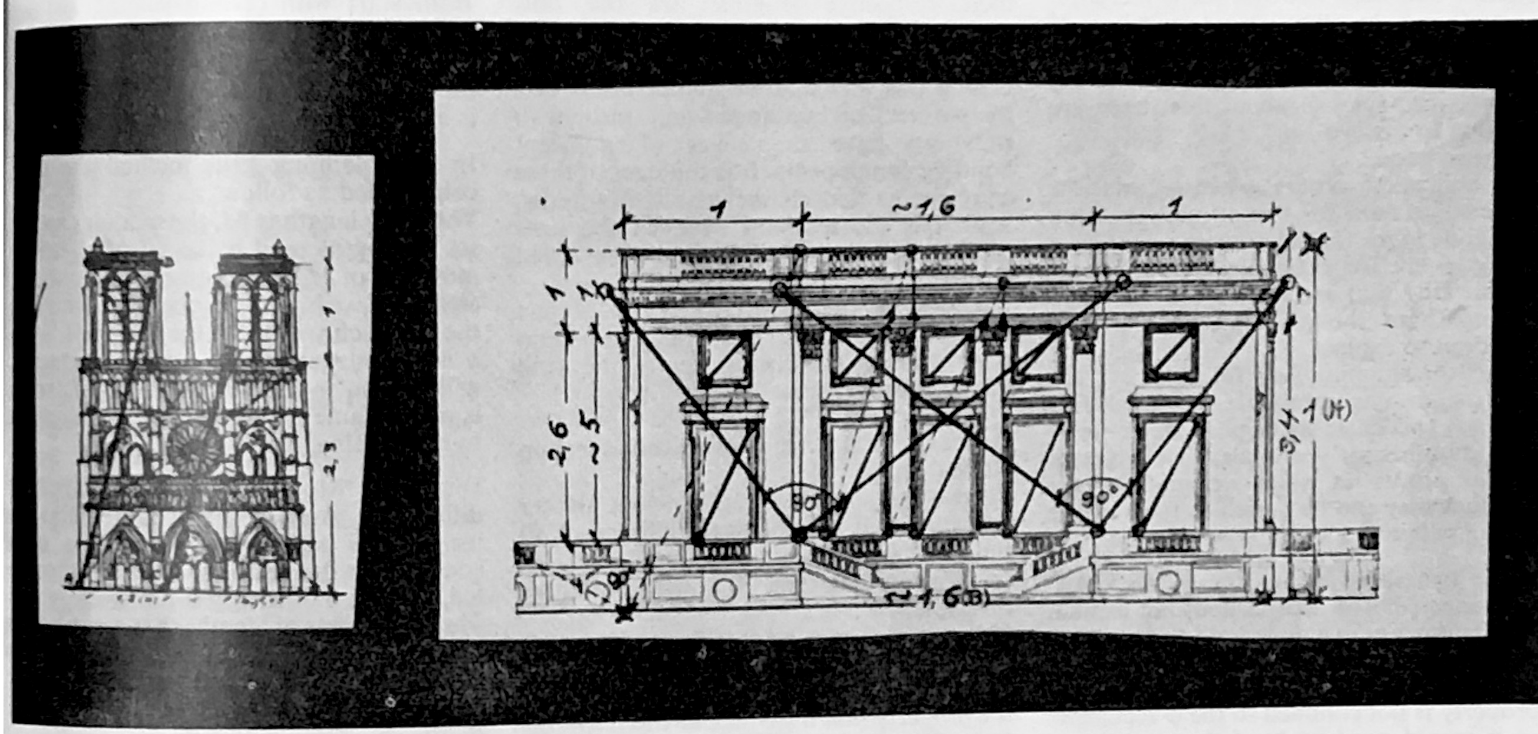

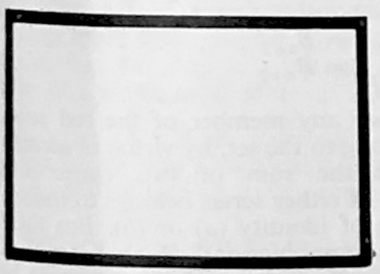

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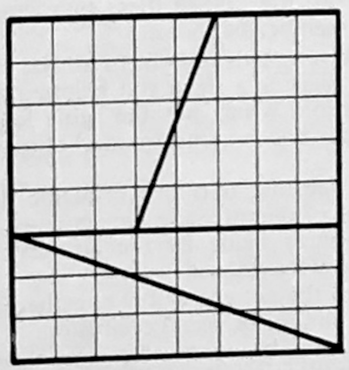
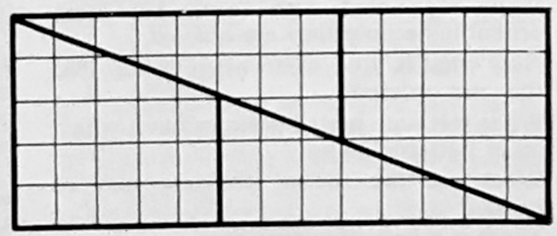
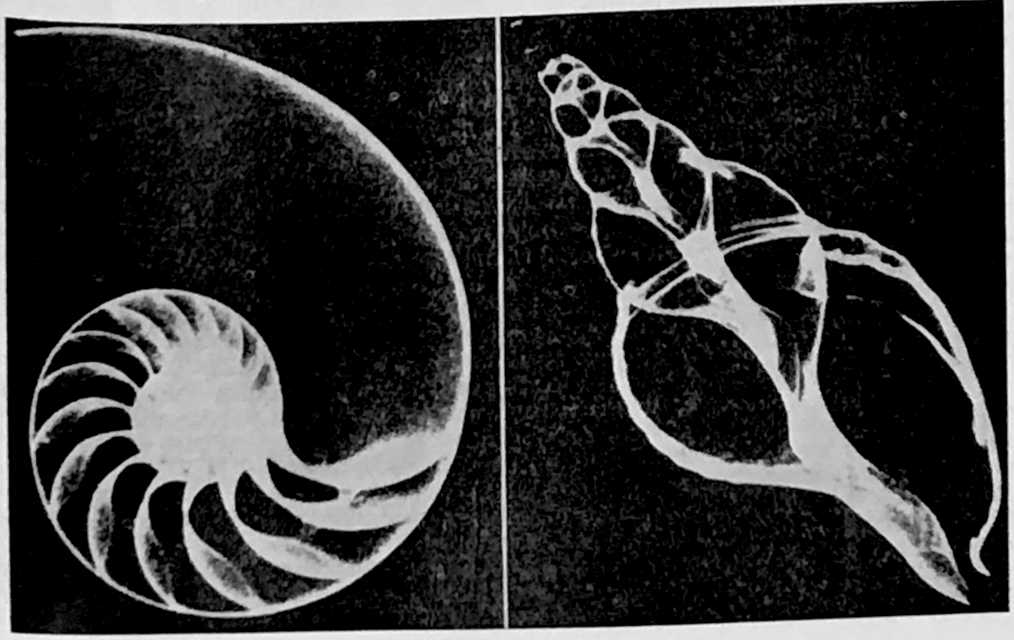


Fig. 4.





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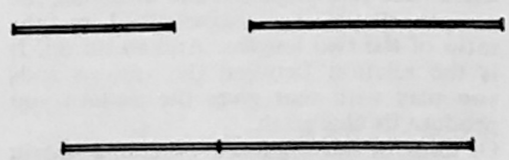

Fig. 6. Given any two rods from the collection, their sum also belongs to the collection.

which we made forms were based on the same principle. And they are not.

A further, quite different, example of confusion is to be found in all the statistical experiments on the golden mean.
A number of rectangles, among them a ϕ rectangle, are shown to people who are asked to say which is the most pleasant.(16)
Often it does turn out that the ϕ rectangle is preferred by the largest number of people—which shows that, within very broad limits (owing to the bounds of visual acuity), rectangles of about this shape are pleasing to the eye.
So far so good.
The confusion occurs when efforts are made to account for this attractiveness by appealing to the formal properties of ϕ (like $1 + \phi = \phi^2$, for example). The 'reasons' of this kind that are offered are far more obscure than the phenomenon they are called on to explain.
In fact the situation is as follows.
1. It is an unaccountable empirical fact that *this* particular shape pleases the eye.
2. A number of rectangles have certain formal properties which account for the fact that they can be nested to form rather simple patterns.

These two facts are, as far as we know, unconnected. The second does not explain the first (except in a disorderly and mystical fashion), and moreover there are all sorts of rectangles that can be nested—the property is not confined to the ϕ rectangle, as it would need to be if there *were* any connection between 1 and 2.
However, these nested patterns *are* visually agreeable, for a different reason. They please the eye, not because of any special shape associated with them, but more probably because they are *ordered*.
Now what is it we mean when we say that they are ordered?
What is it such nested patterns have, which other patterns do not?
What are the special characteristics of order?
There is a lack of confusion.
A certain simplicity.
Relations between the parts.
These are the traditional ways of defining order, aren't they? Let's not scorn them, but try to see what the definitions say.

Essentially a pattern of this kind is characterised by the lengths of the lines that make it up. Imagine yourself making such a pattern from a number of straight rods. If all the rods were of the same length, the pattern would have a very obvious character, and an obvious kind of order too. If there were two lengths of rod available, the pattern would be characterised by the ratio of the two lengths. And so on up. It is the relation between the various rods you play with that gives the pattern you produce its character.
Or think of starting at the other end: if you were allowed rods of as many different lengths as you liked, and used a great variety, the patterns you produced would have no order at all—over and above their rectilinearity. Gradually, as you reduced the number of different lengths, the patterns would grow clearer and clearer. Rectangles of similar shape would begin to appear, the relations between parts of the pattern would become apparent.

If you are making rectilinear patterns of this kind, the number of different rods, and their respective lengths, are the only variables—they are the only things you can change.
It is in this way that we make patterns on the surfaces of buildings—only instead of rods we have the edges of adjacent building components. It is the sizes of these components that characterise the patterns. And it is the relations between the sizes that is responsible for what we have called 'order'.
But what do the relations need to be; just how should we control the component sizes so as to produce order of the kind that is effective, visually?

We can't, of course, answer this question altogether.
But we can suggest an answer—an answer that turns out to explain the efficacy of all the order systems in current use.
It depends on the following additive principle.
Think of the set, S, of different component dimensions (in our example, the set of different lengths of rod). And give this set the property that if we take any two lengths from the set, the *sum* of these two lengths is also a member of the set.(17)
(If we take any two rods from all the rods we have, there is a third rod belonging to the collection, which has the same length as the other two put end to end.) (Fig. 6.)

We shall see first that this principle does meet some of the rather vague demands for satisfactoriness made by the accounts of perception we discussed:
The bits of the pattern will fit together in a way that minimises visual confusion.
Any length can be expressed in several ways as the combination of smaller lengths.
Combinations and their variants recur throughout the pattern.
Relations between different lengths will be apparent just on account of the frequency of their occurrence.
Bits of the pattern will appear several times, arranged and rearranged.
The relations between bits, and the re-appearance of similar bits, will make the whole pattern easy to grasp, easy to recognise.
All these facts will contribute to the well-being of the lazy perceptual mechanisms.
That is as much as we can say.

We cannot be certain that order is pleasing to the eye. But we believe it to be so, and the belief does not seem altogether foolish. When we examine order systems, we shall find that they have the property discussed, so we can say, at least, that *if* they are visually effective, it may well be on account of this property.

The two most prominent systems of recent years have been 'Le Modulor' devised by Le Corbusier, and the 3 ft. 4 in. and 8 ft. 3 in. planning grids used by the Hertfordshire school designers after the war. Both were concerned with the actual lengths that appeared in the building's components, so that, ultimately, these components might be standardised.
Both start with certain lengths, and base on them a set whose members fulfil the condition specified: 'That if any number of lengths are added together, their sum is also a length from the set.'

In the planning grid method the set is constructed as follows:
The basic length is M, the module. And the set of lengths used is the set of all integral multiples of M, which satisfies the condition, clearly. If we have a number of lengths from the set, each will be of the form nM, where n is an integer. Their sum, consequently, will be $(n_1 + n_2 + n_3 \ldots + n_K)M$, which is of the same form, and therefore belongs to the set also.

With the 'Modulor' the set is arranged differently. And we shall find that while its lengths satisfy the condition often enough for the relations between them to tell, they do not satisfy it always.
The set consists of lengths that are members of two interlocking Fibonacci series, the blue series and the red series, which we may denote by:
$B_1, B_2, B_3, B_4, \ldots$ and $R_1, R_2, R_3, R_4, \ldots$
where

$$B_n + B_{n+1} = B_{n+2} \quad \ldots \quad (a)$$
$$R_n + R_{n+1} = R_{n+2} \quad \ldots \quad (b)$$
$$2R_m = B_m \quad \ldots \quad (c)$$

Now twice any member of the red series does belong to the set, by virtue of identity (*c*). And the sum of two consecutive members of either series belongs to the set, by virtue of identity (*a*) or (*b*). But there are many combinations which lie outside the set, and sooner or later we shall be forced to use such combinations, as Le Corbusier himself has been forced to do.(18)

But, essentially, both systems are sets of numbers with what we have called the additive property—a property that can be written down quite simply:
S is a set of numbers such that if x and y both belong to S, then $x + y$ also belongs to S. It can easily be shown that such a set is uniquely defined by its lowest two members, a and b, say; and that all other members of the set may be expressed as linear combinations of a and b—that is, in the form $na + mb$, where n and m are whole numbers greater than or equal to zero.
Consider, for example, the following set:
7 11 14 18 21 22 25 . . .
This set, which fulfils the condition of additivity, may be defined by the pair 7, 11—for:
$14 = 7 + 7, 18 = 7 + 11, 21 = 7 + 7 + 7, 22 = 11 + 11, 25 = 7 + 7 + 11$, etc.
The set contains all possible linear combinations of 7 and 11.
It seems then, that the order produced by such a set is entirely dependent on a pair of numbers (lengths) *a* and *b*.

So we could equally well define the system by specifying a and r, r being the ratio b/a. And indeed, it is in this form that the theory proves most interesting, for, by giving the parameter r different values we can reduce the general system to particular ones, some of which we know already.

If we put $r = \phi$, or $b = \phi a$, and remove several members of the ensuing set of linear combinations, we are left with the double Fibonacci series of the Modulor. The initial set is:
$a \quad \phi a \quad 2a \quad (1 + \phi)a \quad 3a \quad 2\phi a$
$(2 + \phi)a \quad (1 + 2\phi)a \ldots.$ which, since $1 + \phi = \phi^2$, $1 + 2\phi = \phi^3$, etc., we may rewrite as:
$a \quad \phi a \quad 2a \quad \phi^2 a \quad 3a \quad 2\phi a \quad (2 + \phi)a$
$\phi^3 a$ from which we select $a \quad \phi a$
$\phi^2 a \quad \phi^3 a \ldots.$ and $2a \quad 2\phi a$
$2\phi^2 a \ldots.$, which are Le Corbusier's red and blue series.

If we put $r = \sqrt{\phi}$ we arrive at the system underlying the ground plan of Mies' Farnsworth house. For here the set of linear combinations is:
$a \quad \sqrt{\phi}a \quad (1 + \sqrt{\phi})a \quad 2a \quad 2\sqrt{\phi}a \ldots.$, which contains all the lengths that occur in the basic plan, not because Mies chose these lengths numerically, but because the plan is constructed in a certain additive way that has the same effect.

Finally, if we put $r = 1$ or $a = b$, we arrive at the familiar modular planning grid. The set of linear combinations is nothing more than:
$a \quad 2a \quad 3a \quad 4a \quad 5a \ldots.$, the set of multiples of a single module a.

Each value of r, then, gives us a different system—each one a particular example of the general system we have proposed.

An aspect of these systems that we have not yet examined is their scale. We have said nothing of the units to be associated with such sets—for clearly we can assign any units we please to their members. They can be millimetres, inches, feet, miles. We must decide what sort of units will best achieve the purpose under discussion—that of making visually effective patterns.

a and b must not be too large—or the lengths would be too great for us to perceive the relationships between them. (Moving round a building, the eye most readily picks up lengths between 3 ft. and 10 ft.—the scale of windows, doors, panels, floor to ceiling heights, and so on.)
But what is not so clear is that a and b must not be too small either. This depends on the fact that the eye is only able to grasp a very limited range of relationships. We do not see 20 objects as 20, but as a large number. And we see no relation between a length of $19k$ and one of $20k$ (whatever the units). They appear simply as two almost equal lengths—the relation is too obscure for the eye to pick it up.
If our basic lengths were 4 in. and 7 in., for example, the set would contain: 4 in., 7 in., 8 in., 11 in., 12 in., 14 in., 15 in., 16 in., 18 in., and every succeeding inch, so, as we have seen, the eye would notice no relations among the higher members of the set.
a and b will need to be as large as possible, therefore—of the order of three feet, probably.

The final choice of a and b will be determined by their HCF. (The HCF of 3 ft. and 5 ft. being 1 ft., that of 3 ft. and 5 ft. 4 in. being 4 in., that of 3 ft. and 4 ft. 10½ in. being 4½ in.)
When we select the set of linear combinations of a and b, we are in fact picking out certain lengths, which we believe will be visually effective in combination, from all the multiples of their HCF.
For, if this HCF is k, then:
1. *Any* member of the set is a multiple of k.
2. But the set does *not* contain all multiples of k.

It happens that modern technological theory is also based on a small module like our k, and we shall examine the possibility of making k the same as this manufacturers' module, which has been fixed, after considerable research, at 4 in. (approximately 10 cm.).[19]
In what follows it is essential to distinguish clearly between the two aspects of such a modular theory—between the economic approach of the manufacturer and the aesthetic approach of the architectural theorist. From the manufacturer's point of view it is desirable to provide the architect with as big a range of sizes as economics allow. The module is introduced only as a standard that will allow all the parts made by different manufacturers, in different areas, for different purposes, to be used successfully in conjunction with one another. Any interest the manufacturer has in selecting particular lengths from the set of multiples, is governed by his desire to cover the maximum number of modular spaces with the minimum number of elements. He will choose certain multiples of k (like our a and b), and find out what ranges of the set of all multiples can be covered by combinations of them. This problem has been examined recently, with the help of a well-known number theorem which states that *every* multiple of k above $(a - k)(b - k)$ can be expressed as a linear combination of a and b.[20]

The architect who wants to introduce some kind of visual order is concerned with almost the opposite problem. He is concerned with the lengths *below* $(a - k)(b - k)$.
The visual effect associated with a number pair and its set is powerful just because, below $(a - k)(b - k)$, we *cannot* cover all multiples of k with linear combinations of a and b. We can only cover a limited set of these multiples, and it is this very limitation that makes the pattern tell.

In spite of this difference the two views of the situation are quite compatible.
The 4 in. module is enough to make them so.
The manufacturer will select his component sizes on economic grounds, and then, for a particular building, the designer will choose *from these components* a set that has the restriction discussed. (a and b may be chosen from any of the components available—their HCF will always be 4-in., or a multiple of 4 in., because of the 4 in. module used by the manufacturer.)
In this way it will be possible to use the order principle.

The connection between perception and modular co-ordination that has been established could influence a practising designer, certainly. It could be made use of as a tool. But the connection was established, principally, in the belief that we should know what we are up to. And in the hope that it will increase our confidence in modular co-ordination.

It indicates that *if* the various technical organisations succeed in their aim, establish the 4-in. module, and the manufacturers adopt it, we shall be able to design, using this 4-in. module and the traditional principle of order.

And our faith in the visual order we produce will no longer need to be mysterious, but may be in some measure understood.

NOTES

(1) See, for example, Plato's, *Theaetetus*, *Timaeus*, *Philebus; Architectural principles in the age of humanism*, by Rudolph Wittkower, London 1952, throughout.
Gothic cathedrals and sacred geometry, by G. Lesser, London 1957.
(2) For this account see *Principles of Gestalt psychology*, by K. Koffka, London 1935; for its use in art criticism see *Art and visual perception*, by Rudolph Arnheim, London 1956.
(3) The account of D. O. Hebb is contained in his *The organisation of behaviour*, New York 1949.
(4) See *Some informational aspects of visual perception*, by F. Attneave, in PSYCHOLOGICAL REVIEW 61, 1954, pp. 183–193.
(5) This implication is found throughout the writing of W. H. Goodyear, as, for example, *Greek refinements*, Yale U.P. 1912.
(6) We find this in most writing on Eastern architecture: for instance—*India*, by Richard Lannoy, Norwich 1955, p. 8.
The lesson of Japanese architecture, by Jiro Harada, London 1936, pp. 45–51.
(7) *Le modulor*, by Le Corbusier, Boulogne 1950, pp. 233–234.
(8) *Geometrical composition and design*, by Matila Ghyka, London 1952, p. 7.
(9) *Dynamic symmetry*, by Jay Hambidge, Yale U.P. 1920.
(10) $\phi = \frac{1 + \sqrt{5},}{2}$ is the limit of ratios of consecutive members of the Fibonacci series.
(11) The result of some unpublished experiments done at the Building Research Station.
(12) See, for example, *The geometry of art and life*, by Matila Ghyka, New York 1946, Chapter 6.
(13) *Growth and form*, by D'Arcy Wentworth Thompson, Cambridge 1952, pp. 912–933.
(14) See a paper by P. G. Tait, PROCEEDINGS of the Royal Society of Edinburgh, VII, 1872, p. 391.
(15) See note (12). It is never put quite so naïvely, but the implication is always there.
(16) Such experiments were done, for example, by G. T. Fechner who reported them in *Vorschule der Aesthetik*, Leipzig 1876, pp. 190–202. More recently by T. R. Austin and R. B. Sleight, JOURNAL OF APPLIED PSYCHOLOGY, 35, 1951, pp. 430–431.
(17) This is the first and most important axiom that defines a *commutative additive group*, in algebra.
(18) To construct a satisfactory bay width at Marseilles, for instance, he adds 53 cm. to 366 cm., making 419 cm., which does not belong to either series. See *Modulor* 2, London 1958, p. 237.
(19) See *Modular co-ordination in building*, published by the European productivity agency, Paris, August 1956.
(20) *Geometrical aspects of modular co-ordination*, by J. W. Harding and L. S. Vallance, THE BUILDER, 27 Sept. 1957, pp. 552–555.